# Load Modeling For Power System Analysis

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*Abstract*—This paper covers overview of the load models for distribution systems. The basic concepts for static, dynamic and combined models are introduced. The following discussion shows importance of accurate system parameters modeling for analysing power system stability. The accurate load representation is not easy issue regarding to its changeable nature and variety. Basic load classification is described.

### I. INTRODUCTION

Power system stability and reliability have been very important issues over the years. It is hard to predict the

system behaviour in case of outage, voltage or frequency variation or other factors that can affect system stability without proper information, or system behaviour data. The powers system are mostly nonlinear and operates in constantly changing conditions, so it is hard to obtain precise information and data about i.e. generators or loads during disturbance[2]. This required to either assume or measure certain quantities like real and reactive power, to predict system response to abnormal conditions. Very

important factor is to not interrupt power flow to the residential, industrial or any other end users. That is why certain measurements need to be made with specialized, expensive equipment to allow get the data under operating system. This type of data collecting involves a continuous monitoring process and requires data to be processed after the measurements are done.[3]. A lot of studies have been carried out so far to provide as accurate model of load as possible, and still new techniques are being developed through big electric concerns. There is a lot of computer software existing that allows for load modeling and system behaviour simulation, however, as mentioned above, the more accurate data typed in to the program, the more accurate model and the simulation results."It has become clear that assumptions regarding the load model can impact predicted system performance as significantly as the models chosen for excitation systems and synchronous machines, which have received greater industry attention"[5]

"Analysis of power system dynamic performance requires the use of computational models representing the nonlinear differential–algebraic equations of the various system components. While scale models or analog models are sometimes used for this purpose, most power system dynamic analysis is performed"[2].

## II. LOAD TYPES AND CLASSIFICATION

Before load modeling process will get started, it is required to understand its applications that are divided into two categories: static, called "snap-shot" – with respect to time as well as dynamic – time varying. Since the "static model based

on the steady state method of the network representation in the power flow networks"[1], these models express loads as a voltage magnitude function. The dynamic model, in turn, represents variation of the load with the frequency. The fundamental starting point for the load modeling is at the distribution level, thus, the applications outside of power system can be as follows:[1]

*Static applications* – this model considers only voltage – depended characteristic[1]:

- Power Flow (PF)
- o Harmonic Power Flow (HPF)
- o Transmission System (TPF)
- o Transmission Power Flow
- Voltage Stability

*Dynamic applications* –considers both voltage-dependent and frequency dependent characteristics[1]:

- Transient Stability (TS)
- Dynamic Stability (DS)
- Operator training simulators (OTS)

As one already mentioned, the loads are quite difficult to be modeled in terms of their variety and change in load depending on the time of the day or season. It is necessary to classified them according to particular area, composition of each one as well as regarding to load characteristic. The loads can be also combined in groups and exhibited as: industrial, residential, commercial and agricultural[3]. The industrial loads includes mainly induction motors, up to 95%, the residential loads includes most of domestic appliances, i.e. refrigerators, washing machines etc. as well as heating and air conditioning units. The commercial load is refers to air condition units and discharge lighting in particular, whereas agricultural loads corresponds to induction motors, i.e. as a prime movers for pumps.[3] Having presented load classification and general load requirements, it is possible now, to carry out the theoretical analysis to express the system load as a mathematical models. In order to do this, one needs to use some mathematical expressions to describe the load as accurate as possible, what is described in the next section of this paper.

#### **III. LOAD MODELING CONCEPTS**

Generally, one can distinguish tow basic load modeling concepts: component based and measurement based models. Component – based model is built on the base of the information on each elements that consist of the load being simulated. "Each load component type is tested and to determine the relationship between real and reactive power requirements versus applied voltage and frequency."[1], then the load model is developed mainly in either exponential or polynomial form basing on obtained data."The range of validity of each model is directly related to the range over which the component was tested"[1]. Usually, the load model is expressed on the p.u values for convenience. By combining appropriate load model types one can approximate the composite load. Mentioned load model types are obtained basing on the load survey information.[1] The composition of the load depends upon the time of the day, moth, or season as well as atmospheric conditions. One can notice, that the country with fine weather revel higher demand for air conditioning, while in the cold climate countries it is just opposite, there is a big demand for electricity heating. One can point out, that these demand can also vary seasonally, in the summer/winter periods, demand for heating/cooling can either increase or decrease.[3] Having introduced the model types and its variation depending on certain factors, one can propose some mathematical load model representation as a set of equations in order to give details of the relationship between the input and output system parameters. Considering load modeling, mathematical representations is related to voltage and/or frequency measured at the buses as well as real and reactive power consumed by the loads.[3]

#### IV. LOAD MODELING

"For dynamic performance analysis, the transient and steady-state variation of the load P and Q with changes in bus voltage and frequency must be modeled" [5]. For power system analysis load can be thought as real and reactive power launched to lower voltage distribution system at buses represented as network model. Among lots of devices and appliances being connected to the system and considered as a load, one should include intervening distribution systems feeders, shunt capacitors, transformers etc. as well as voltage controlling devices or voltage regulators. The simplest load model is *static model*. Next chapter introduces static models as well as dynamic ones that are being used for i.e. system stability studies.

#### V. DELTA AND WYE CONNECTED LOADS

First load models to be described are Wye and Delta connected loads. Each of these loads can be represented as being connected phase-to-neutral or phase-to-phase in a four wire Wye system or phase-to-phase in a three wire delta system[4]. All of model above can be three, two, or single phase and modeled as: constant real and reactive power, constant current, constant impedance as well as any random combination of the above. This model is one of the basic static load models.

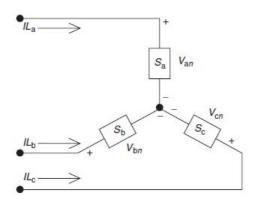


Fig.1 Wye connected load[1]

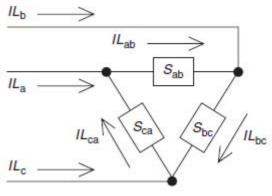


Fig.2 Delta connected load[1]

Both load models depicted in the figures above require the load component current coming into the load to be determined.[r4]. Assumption is made, that the loads have an initial complex power

$$|S|_{a,b,c} \angle \theta = P_{a,b,c} + jQ_{a,b,c}$$
(1)

, where indices a,b,c denotes that formula is expressed in the same way for each single phase of Wye connected load, this notation will be kept in further formulas describing model above

A. Constant Real and Reactive power loads for Y connection

"In this model the line to neutral voltage will change during each iteration until convergence is achieved"[4]

$$IL_{a,b,c} = \left(\frac{S_{a,b,c}}{V_{(a,b,c)n}}\right)^* = \frac{|S|_{a,b,c}}{V_{(a,b,c)n}} \angle \delta_{a,b,c} - \theta_{a,b,c} = \angle \alpha$$
(2)

### B. Constant Impedance Load for Y connected loads

Constant impedance is first determined from the specified complex power and line-to-neutral voltage for each phase.

$$Z_{a,b,c} = \frac{\left| V_{(a,b,c)n} \right|^2}{S_{a,b,c}^*} = \frac{\left| V_{(a,b,c)n} \right|^2}{\left| S_{a,b,c} \right|} \angle \theta_{a,b,c} = \left| Z_{a,b,c} \right| \angle \theta_{a,b,c}$$
(3)

Having "constant load impedance", it is possible to express load currents as the function of those impedance. This current for each phase is given by equation (4):

$$IL_{a,b,c} = \frac{V_{(a,b,c)n}}{Z_{a,b,c}} = \frac{|V_{(a,b,c)n}|}{|Z_{a,b,c}|} \angle \delta_{a,b,c} - \theta_{a,b,c} =$$
$$|IL|_{a,b,c} \angle \alpha_{a,b,c} \quad (4)$$

"In this model the line to neutral voltage will change during each iteration until convergence is achieved"[4]

C. Constant Real and Reactive power loads for Y connection

$$IL_{a,b,c} = IL|_{a,b,c} \angle \delta_{a,b,c} - \theta_{a,b,c}$$

For this model, current magnitudes are obtained according to the equation (2). These magnitudes do not change until the voltage angle  $\delta$  changes during each iteration[4],

#### D. Delta connected load model

Similarly to the Y connected load, the delta connected load model can be made. The equations describing this model are the same, except that voltages and currents in this models are taken as phase-to-phase instead of phase-to-neutral, so delta load model is described by following equations:

Constant real and reactive power:

$$IL_{x} = \left(\frac{S_{x}}{V_{x}}\right)^{*} = \frac{|S|_{x}}{V_{x}} \angle \delta_{x} - \theta_{x} = |IL_{x}|\delta_{x} (6)$$

Constant Impedance Load

$$Z_{x} = \frac{|V_{x}|^{2}}{S_{x}^{*}} = \frac{|V_{x}|^{2}}{|S_{x}|} \angle \theta_{x} = |Z_{x}| \angle \theta_{x}$$
(7)

Constant Current Load

$$IL_{a,b,c} = \frac{V_x}{Z_x} = \frac{|V_x|}{|Z_x|} \angle \delta_x - \theta_x = |IL|_x \angle \alpha_x$$
(8)

, where index x denotes line-to-line values ab, bc, and ca respectively.

Models presented above can be modeled as combination of above constant Z, I, and P and can be assigned as a percentage of total load shown above. For the Wye connection, it is simply the sum of the three components Ia, Ib and Ic, while for all delta connected loads the line current is determined by:

$$\begin{bmatrix} IL_{a} \\ IL_{b} \\ IL_{c} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} IL_{ab} \\ IL_{bc} \\ IL_{ca} \end{bmatrix}$$

Considering Wye and Delta connected loads one can mention of the Shunt Capacitor models that are very often used for voltage regulation in a distribution system as well as in order to provide reactive power support.

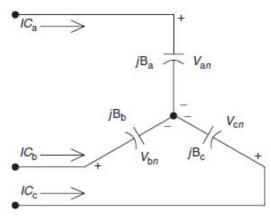


Fig. 3 Wye connected capacitor bank[1]

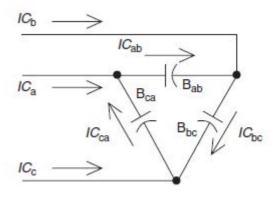


Fig. 4 Delta connected capacitor bank[1]

The capacitor banks are modeled as constant susceptances connected as above. The susceptance can be found as:

$$B_{actual} = \frac{kVAr}{kV^2 1000} [Siemens] \quad (10)$$

$$B_{actual} = \frac{kVAr}{kV^2 1000} [ner unit] \quad (11)$$

where 
$$\frac{kVAr_{p,u}}{kVA_{zimula}}$$
 (12)

$$V_{p.u} = \frac{kV_{actual}}{kV_{line \ to \ neutral \ base}}$$
(13)

Having computed susceptance one can find out the line currents given by:

$$IC_a = jB_a V_{an} \quad (14)$$
  

$$IC_b = jB_b V_{bn} \quad (15)$$
  

$$IC_c = jB_c V_{cn} \quad (16)$$

For delta connection currents flowing in capacitors are given as below:

$$\begin{array}{ll} IC_{ab} = jB_{ab}V_{ab} & (17) \\ IC_{bc} = jB_{bc}V_{bc} & (18) \\ IC_{ca} = jB_{ca}V_{ca} & (19) \end{array}$$

The line currents feeding the delta connected capacitor bank are as follows:

$$\begin{bmatrix} IC_a \\ IC_b \\ IC_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} IC_{ab} \\ IC_{bc} \\ IC_{ca} \end{bmatrix}$$
(20)

# VI. POLYNOMIAL AND EXPONENTIAL LOAD MODELS

There are two mathematical forms that are commonly used in modeling studies: polynomial and exponential load models. Polynomial model is a static model describing relationship of the power and voltage magnitudes as a polynomial equation and can be expressed as[6]:

$$P = P_0 \left( a_1 \left( \frac{V}{V_0} \right)^2 + a_2 \left( \frac{V}{V_0} \right) + a_3 \right) (21)$$
$$Q_L = Q_0 \left( a_4 \left( \frac{V}{V_0} \right)^2 + a_5 \left( \frac{V}{V_0} \right) + a_6 \right) (22)$$

,while exponential model has the form of:

$$P = P_0 \left(\frac{V}{V_0}\right)^{np} (23)$$
$$Q = Q_0 \left(\frac{V}{V_0}\right)^{nq} (24)$$

"The parameters of this model are exponents, and the power factor of the load"[6]. Setting these parameters to 0,1, or two the load model becomes constant current, power, or impedance, like in case of the Wye and Delta connected loads shown in previous section.[6]

# A. PSS/E Static Model[12]

This static model introduces the active and reactive power that are both frequency and voltage dependent and given by the following formulas:

$$P_{L} = P_{0} \left( a_{1} \left( \frac{V}{V_{0}} \right)^{n1} + a_{2} \left( \frac{V}{V_{0}} \right)^{n2} + a_{3} \left( \frac{V}{V_{0}} \right)^{n3} \right) (1 + a_{7} \Delta f)$$
(25)

$$Q_{L} = Q_{0} \left( a_{4} \left( \frac{V}{V_{0}} \right)^{n4} + a_{5} \left( \frac{V}{V_{0}} \right)^{n5} + a_{6} \left( \frac{V}{V_{0}} \right)^{n6} \right) (+a_{8}\Delta f)$$
(26)

,"where  $a_1$ -  $a_8$  and  $n_1$ -  $n_8$  – parameters to be estimated when  $V=V_0$ 

#### VII. CONCLUSIONS

The both static, and dynamic model of loads were described in this work as well as combined ones. Load modeling is a challenge for today's engineers. It's variety and not constant nature makes them hard to simulate. There have been a lot of studies on load modeling in order to represent it with maximum accuracy. There are many simulation software that allows to simulate power system loads basing. i.e. on polynomial and exponential load model representation.

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