# Disturbances correlated to compensation of reactive power in system with wind generation – estimation of transient components' parameters

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*Abstract*—precise computation of current components is a key prerequisite for reliable assessment of power quality. Especially in networks with wind generation we may observe increased number of possible disturbing phenomena. This paper presents an approach to accurate computation of currents components with two similar parametric methods based on singular value decomposition (SVD) and Prony model. Those methods seem to be applicable for the detection of non integer multiples of the main frequency in decaying signals. Results of both methods have been compared and evaluated. with respect to traditional Fourier approach.

*Index Terms*—power quality, transients, SVD, Prony method.

# I. INTRODUCTION

The widespread implementation of wind energy conversion systems is a reality. Wind is seen as a clean and renewable energy source, so the development of wind generation technologies is welcomed and supported by ecologists and governments. In the next years we will have even more generator units connected to the grid [1].

Wind turbines, despite of their advanced control systems and power electronic converters, influence in many different aspects the electrical system they are connected to [2, 3]. So far, the electrical distribution networks were designed and operated under the assumption of centralized generation and an energy flow from the substation to the consumer. It is no more the case [3, 4]. The connection of wind generators could lead to many disturbances, such as: voltage fluctuations, flickers, harmonics, instability, blind power regulation problems, and transients [10]. Power quality issues connected with wind generation are not only important because of technical aspects, they are also crucial on the free energy market.

There are at least three main wind generators structures, which can be easily pointed out [4]. The simplest and previously popular is the squirrel-cage induction generator connected directly to the grid. Usually, that type of turbines has a fixed pitch of turbine blades. Second is the doubly-fed induction generator. The stator winding of this generator is coupled with the system grid, and the rotor winding is connected to a voltage-source converter. The converter adjusts the frequency of the rotor feeding current in order to enable variable speed operation. The wind generator operates in wide spectrum of wind speeds and has lower impact on the grid, but the investment costs are higher. The third structure of wind generation unit has a synchronous machine. The rotating shaft and generator are coupled directly without gear box. That generators type requires a back-to-back converter for the grid connection, but it can be operated in wide wind change range. Additionally, voltage, active and reactive power can be controlled, as in double feed induction generator.

Many of the wind energy converters installed today still have a squirrel-cage induction machine connected directly to the grid [6, 7]. This type of the generator cannot perform voltage control and it absorbs reactive power from the grid. Phase compensating capacitors are usually directly connected. That type of wind turbine is cheap and robust and therefore popular, but from the system analysis point of view it has some drawbacks [4].

An important disadvantage is that during the switching of the phase compensating capacitors, transients occur [7], which can be disturbing for sensitive equipment, protection relays and insulation. Also the impact on power quality indices can not be neglected [5, 10].

Transient overvoltages can theoretically reach peak values up to 2.0 pu. High current transients can reach values up to ten times the nominal capacitor current with a duration of several milliseconds [8].

The purpose of this paper is the assessment of transients in electrical system for wind turbines equipped with an asynchronous generator. A wind energy converter connected to a distribution system was modeled in Matlab SimPowerSystemsTolbox [9].

The Prony model and SVD method were considered as appropriate tools for the parameters estimation of transients. The analysis was carried out for different operation conditions of the wind energy converter.

# II. COMPUTATIONAL METHOD FOR SPECTRUM ESTIMATION BASED ON SINGULAR VALUE DECOMPOSITION (SVD)

The proposed method, based on Singular Value Decomposition (SVD) [14, 15, 16], is a technique for modeling sampled data as a linear combination of exponentials.

Accordingly, the signal x(t) can be described mathematically as a sum of N exponential components:

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \dots + A_N e^{s_N t}$$
(1)

where: A - signal amplitude,  $s = \alpha + j\omega$  - complex frequency.

For practical reasons we consider only digital signals, sampled with the interval T

$$t = [nT]$$
(2)  
The signal samples are given by

$$x_{n} = A_{1}(e^{s_{1}T})^{n} + A_{2}(e^{s_{2}T})^{n} + \dots + A_{N}(e^{s_{N}T})^{n} = A_{1}z_{1}^{n} + A_{2}z_{2}^{n} + \dots + A_{N}z_{N}^{n}$$
(3)

The practical realization of the proposed method imposes designing of a digital filter with finite impulse response (FIR). This filter (Fig. 1) should block all signal components. The filter transmittance in general form is given by

$$H(z) = 1 + h_1 z^{-1} + h_2 z^{-2} + \dots + h_N z^{-N}$$
(4)



Fig. 1. Basic digital filter scheme as a practical realization of proposed analysis method

The filter response is given by

$$Y(z) = H(z) \cdot X(z)$$
<sup>(5)</sup>

The blocking property of the considered filter results in

$$y_N = x_N + h_1 x_{N-1} + h_2 x_{N-2} + \dots + h_N x_0 = 0$$
  
$$v_n = 0 \quad for \ n \ge N$$
(6)

Equation (6) can also be written in a practice relevant matrix form

$$\begin{bmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{N-1} \\ x_{N} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -h_{N} & -h_{N-1} & -h_{N-2} & \cdots & -h_{1} \end{bmatrix} \begin{bmatrix} x_{0} \\ x_{1} \\ \vdots \\ x_{N-2} \\ x_{N-1} \end{bmatrix}$$
or
$$(7)$$

$$\mathbf{x}_1 = \mathbf{S}\mathbf{x}_0 \tag{8}$$

Generally, the realization of proposed method implies using *M*-column matrices instead of one column vectors of input signal samples  $x_n$ .

$$\mathbf{X}_{1}^{(M)} = \mathbf{S}\mathbf{X}_{0}^{(M)} \tag{9}$$

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The construction of the  $\mathbf{X}_{1}^{(M)}$  matrix is given in (10).

The 
$$\mathbf{X}_{0}^{(M)}$$
 matrix is constructed in a similar manner.

$$\mathbf{X}_{1}^{(M)} = \begin{bmatrix} \mathbf{X}_{1} & \mathbf{X}_{2} & \mathbf{X}_{3} & \cdots & \mathbf{X}_{M-1} & \mathbf{X}_{M} \\ \mathbf{X}_{2} & \mathbf{X}_{3} & \mathbf{X}_{4} & \cdots & \mathbf{X}_{M} & \mathbf{X}_{M+1} \\ \mathbf{X}_{3} & \mathbf{X}_{4} & \mathbf{X}_{5} & \cdots & \mathbf{X}_{M+1} & \mathbf{X}_{M+2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{X}_{N-1} & \mathbf{X}_{N} & \mathbf{X}_{N+1} & \cdots & \mathbf{X}_{N+M-3} & \mathbf{X}_{N+M-2} \\ \mathbf{X}_{N} & \mathbf{X}_{N+1} & \mathbf{X}_{N+2} & \cdots & \mathbf{X}_{N+M-2} & \mathbf{X}_{N+M-1} \end{bmatrix}$$
(10)

Similarly, equation (3) can also be given in a modified matrix form

$$\mathbf{x}_1 = \mathbf{W}_0 \mathbf{Z} \mathbf{A} \tag{11}$$

$$\mathbf{W}_{0} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ z_{1} & z_{1} & z_{1} & \cdots & z_{1} & z_{1} \\ z_{1}^{2} & z_{2}^{2} & z_{3}^{2} & \cdots & z_{N-1}^{2} & z_{N}^{2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ z_{1}^{N-2} & z_{2}^{N-2} & z_{3}^{N-2} & \cdots & z_{N-1}^{N-2} & z_{N}^{N-2} \\ z_{1}^{N-1} & z_{2}^{N-1} & z_{3}^{N-1} & \cdots & z_{N-1}^{N-1} & z_{N}^{N-1} \end{bmatrix}$$
(12)
$$\mathbf{Z} = \begin{bmatrix} z_{1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & z_{2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & z_{3} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & z_{N-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & z_{N} \end{bmatrix}$$
(13)

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \dots & \mathbf{A}_{N-1} & \mathbf{A}_N \end{bmatrix}^T$$
(14)

Considering (8) and (11) the following equation can be formulated

$$\mathbf{W}_{0}\mathbf{Z}\mathbf{A} = \mathbf{S}\mathbf{W}_{0}\mathbf{A}$$
(15)  
finally

$$\mathbf{S} = \mathbf{W}_0 \mathbf{Z} \mathbf{W}_0^{-1} \tag{16}$$

The latest equations shows, that  $z_1, z_2, z_3, ..., z_N$  are the eigenvalues of the matrix **S**.

Regarding (9) the matrix S may be calculated

$$\mathbf{S} = \mathbf{X}_{1}^{(M)} \mathbf{X}_{0}^{(M)T} (\mathbf{X}_{0}^{(M)} \mathbf{X}_{0}^{(M)T})^{-1}$$
(17)

Knowing the  $z_1, z_2, z_3, ..., z_N$  values the complex frequencies (3)  $s_1, s_2, s_3, ..., s_N$  may be easily calculated. Similarly the values of amplitudes  $A_1, A_2, A_3, ..., A_N$  may be derived from (11).

### III. PRONY MODEL APPLIED FOR COMPUTATION OF SPECTRAL COMPONENTS IN SIGNALS

The Prony method is a technique for modeling sampled data as a linear combination of exponential functions [8]. Although it is not a spectral estimation technique, the Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics.

Assuming N complex data samples, the investigated function can be approximated by p exponential functions:

$$y[n] = \sum_{k=1}^{p} A_k e^{(\alpha_k + j\omega_k)(n-1)T_p + j\psi_k}$$
(18)

where

n = 1, 2, ..., N,  $T_p$  - sampling period,  $A_k$  - amplitude,  $\alpha_k$  - damping factor,  $\omega_k$  - angular velocity,  $\psi_k$  - initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n] = \sum_{k=1}^{p} h_k z_k^{n-1}$$
(19)

where

 $h_k = A_k e^{j\psi_k}$ ,  $z_k = e^{(\alpha_k + j\omega_k)T_p}$ 

The estimation problem is based on the minimization of the squared error over the N data values

$$\delta = \sum_{n=1}^{N} \left| \varepsilon[n] \right|^2 \tag{20}$$

where

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{p} h_k z_k^{n-1} \quad (21)$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method, that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data can be made.

Consider the *p*-exponent discrete-time function:

$$x[n] = \sum_{k=1}^{p} h_k z_k^{n-1}$$
(22)

The p equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \dots & z_p^0 \\ z_1^1 & z_2^1 & \dots & z_p^1 \\ \vdots & \vdots & & \vdots \\ z_1^{p-1} & z_2^{p-1} & \dots & z_p^{p-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[p] \end{bmatrix}$$
(23)

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the exponents as its roots:

$$F(z) = \prod_{k=1}^{p} (z - z_k) = (z - z_1)(z - z_2)...(z - z_p) \quad (24)$$

The polynomial may be represented as the sum:

$$F(z) = \sum_{m=0}^{p} a[m] z^{p-m} =$$

$$= a[0] z^{p} + a[1] z^{p-1} + \ldots + a[p-1] z + a[p]$$
(25)

Shifting the index on (5) from n to n-m and multiplying by the parameter a[m] yield:

$$a[m]x[n-m] = a[m]\sum_{k=1}^{p} h_k z_k^{n-m-1}$$
(26)

Equation (9) can be modified into:

$$\sum_{m=0}^{p} a[m]x[n-m] =$$

$$= \sum_{k=1}^{p} h_k z_k^{n-p} \left\{ \sum_{m=0}^{p} a[m] z_k^{p-m-1} \right\}$$
(27)

The right-hand summation in (10) may be recognized as a polynomial defined by (8), evaluated at each of its roots yielding the zero result:

$$\sum_{n=0}^{p} a[m] x[n-m] = 0$$
(28)

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots  $z_k$ 

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. N > 2p. In the over determined data case, the linear equation (11) should be modified to:

$$\sum_{m=0}^{p} a[m]x[n-m] = e[n]$$
(29)

The estimation problem is based on the minimization of the total squared error:

$$E = \sum_{n=p+1}^{r} |e[n]|^2$$
(30)

## IV. SIMULATION OF SYSTEM WITH INDUCTION GENERATOR AND CAPACITORS

The wind generator with compensating capacitors is shown in Fig.2. The simulation was done in Matlab using the SimPowerSystem Toolbox [9].



Fig. 2. Induction generator with compensating capacitors.

A wind turbine generates power and accordingly a mechanical torque on the rotating shaft, while the electrical machine produce an opposing electromagnetic torque [4]. In steady state operation, the mechanical torque is converted to real electrical power and delivered to the grid. The power generated by the wind turbine is [4, 6]

$$P = \frac{1}{2}\rho A C_p V^3 \tag{31}$$

and the torque can be found as

$$T = \frac{P}{\omega_s} \tag{32}$$

where  $\rho$  - density of air, A - swept area of the blade,  $C_p$  - performance coefficient, V - wind speed, Tmechanical torque, P - output power of the turbine,  $\omega_s$ rotor speed of the turbine. At the constant wind speed, the  $C_p$  coefficient depends on the rotor speed  $\omega_s$  and pitch angle and is often presented in a table form [10]. The turbine characteristic used in simulation is shown in Fig. 3.



Fig. 3. Induction generator output power vs. angular velocity

The pitch control dynamic can be neglected in power system transient analysis [6]. The simulated generator is a 150 kW, 400 V, 1487 rpm, induction machine. It is connected to the grid through a Dyg 25/0.4 kV distribution transformer which nominal power was varied between 0.5 and 2 MW during the research process and other parameters were set with accordance to [11]. A typical 5 km overhead line [11] connected the generator to a system. The system was represented by equivalent

source with short circuit capacity of 100 MVA and X/R ratio of 7. The induction generator reactive power demand varies with the produced real power [10]. During the research different compensation levels were simulated. Simulation results correlate with measured values [7].

# V. RESULTS OF SIGNAL ANALYSIS

The capacitors indicated in Fig. 2 were build as a bank of separately closed units, what is close to practice [7]. In this practical case two-stage phase-compensating capacitors were connected at the wind turbines. One small (C1=0.5 mF), for low rotor speed and both (C1+ C2=1.7 mF), when the wind turbines operated at higher rotor speed.

Transients occurring during switching of the capacitor bank are shown in Fig. 4. Clearly visible are two stages of switching.



Fig. 4. Currents during two steps of capacitor closing - one phase.

### A. Disturbances due to first capacitor switching

The analysis of transients after the first capacitor are analyzed in this chapter. More detailed waveform of the current is shown in Fig. 5.



Fig. 5. Transient current during the first step of capacitor closing.

Fourier analysis indicated two spectral components (Fig. 6). However, the computation of amplitude of a decaying component is not accurate using Fourier.



Fig. 6. Fourier transform of the current during first capacitor closing

Therefore Prony and SVD approach were used to compute detailed signal parameters.



Fig. 7. First component in the current, reconstructed with Prony method

Fig. 7 and 8 depict the main 50 Hz current component and the switching transient reconstructed with parameters given by Prony model.



Fig. 8. Second component in the current, reconstructed with Prony method.



Fig. 9. Two spectral components computed with SVD method

Similar results have been achieved with SVD. Frequencies indicated by Prony are shown in Fig. 9.

Detailed information of the first and second signal component obtained with Prony and SVD model are given in Table I. The information includes the values of amplitude – A, decaying time constant  $-\tau$ , frequency – f, and initial phase –  $\psi$ .

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CURRENT COMPONENTS A	AFTER FIRST	CAPACITOR	SWITCHING			

Signal com. [No.]	I [A]	τ [s]	f [Hz]	Ψ [rd]			
Prony							
1.	162.9		49.86	0.87			
2.	2225	0.004	592.2	1.5			
SVD							
1.	164,7		49.43	0.90			
2.	2128,9	0.004	588,2	1.52			

# B. Disturbances due to second capacitor switching

Similar procedure was applied for the transient current after closing of the second capacitor (Fig. 10).



Fig. 10. Current after second capacitor closing



Fig. 11. Fourier transform of the current during second capacitor closing

Fourier transform (Fig. 11) of the current in Fig 10 indicates three components.



Fig. 12. First component in the current, reconstructed with Prony method

Fig. 13, 13 and 14 depicts the three components given by the Prony model.



Fig. 13. Second component in the current, reconstructed with Prony method.



Fig. 14. Second component in the current, reconstructed with Prony method.

Similar results were obtained with SVD method (Fig. 15).



Fig. 15. Three spectral components computed with SVD method

Detailed information of the three components obtaiend with SVD and Prony method is given in Table II. The information includes the values of amplitude – A, decaying time constant  $-\tau$ , frequency – f, and initial phase –  $\psi$ .

Signal com. [No.]	I [A]	τ [s]	f [Hz]	Ψ [rd]		
Prony						
1.	51.21		50.97	0.75		
2.	13.84	0.44	349.2	0.30		
3.	831.0	0.004	519.2	1.91		
SVD						
1.	48.85		50.45	0.76		
2.	12.08	0.47	352,11	0.32		
3.	736,8	0.004	512.27	2.01		

### VI. CONCLUSIONS

The research results show, that the Prony model and SVD method are useful for transient estimation in systems with wind generators and compensating capacitors. Those methods enabled accurate estimation of amplitude, time constant, phase and frequency of transients' components of simulated signals. The current waveform and its parameters depend on the capacitor to be switched. Those dependences could be observed during simulation.

Application of both Prony Model and SVD method required a signal model. Fourier transform, as non parametric method, did not require a signal model or even the number of components, but could not compute signal parameters besides frequency. Two signal components were predefined, so the rest was considered noise.

The order of the signal model could be easily extended to detect additional components. Both methods delivered similar results, however, application of Prony model seems more suitable for estimation of signal parameters.

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