Application and Comparison of LQR and Robust Sliding Mode Controllers to Improve Power System Stability

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Abstract— Heffron-Philips model has had great popularity to simulate mono or multi machine power systems. Since the stability of this model is related to operating point of synchronous generator(s), many efforts have been made in research papers to design robust and reliable controllers to ensure the stability of the system. A typical mono-machine power system is presented in this paper to make a comparison between behaviors of traditional power system stabilizer and LQR based pss. Characteristics and fundamental concepts of each controller are stated. At the termination of the paper, Sliding Mode method has been engaged so that the power system can be stable in uncertain condition. The results of LQR base and sliding mode based PSSs have been compared under uncertainty.

Keywords— Heffron-Philips model, pss, stability, LQR.

I. Introduction

Connecting small generation and distribution systems together makes better stability against small disturbances but when a serious problem occurred for one of the connected systems, it also affects the others. These connections have been done between even different countries for economic reasons. Modern power systems have many characteristics such as far distances of consumption from generation declining the power system stability. In order to provide stability for such modern systems, different control systems have been designed and applied in power plants and power transmission lines. Drum level control, Governor, and Automatic Voltage Regulator are such control systems which are applied in power plants. Governor and AVR have bound capability to damp transient state oscillations. Therefore another control system called power system stabilizer, pss, is applied to improve the stability. This additional loop can be applied for both governor and AVR control loops. Applying pss to governor makes many quick variations in the position of mechanical fuel gate making it injured. Hence it is recommended to apply pss to AVR controller [1].

Static Var Compensator, SVC, synchronous condenser, tap changer, and FACTS devices are control systems in power transmission lines.

To simulate and control a generator, it is important to have a model. State space model is a well-known for this aim. The complete model for a synchronous machine has 7 orders. It is efficient to reduce the degree of the system as well as linearization in order to simplify the model. Heffron-Philips model is such a linear model with 3 orders. This model is considered to apply controllers in this paper.

II. Case Study

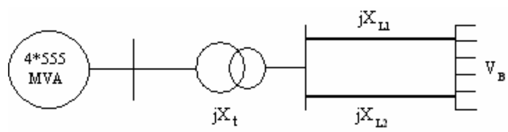

Figure 1. The power system anticipated as case study

Figure (1) indicates a power system as our case study. It includes a power plant comprising four same units. The power plant is connected to a transmission line by a transformer. The line conveys the power to an infinitive bus.

Electrical parameters presented on table (1) are pertaining to normal regime operation of the system [1]. The values are in per unit system.

Table 1. Initial values for the power system

Generator constants	$H=3.5$, $D=0.5$, $T'_{do}=8.0$ $X_d=1.81$, $X_q=1.76$, $X'_d=0.3$
Exciter constants	$K_A=10$
Line and transformer constants	$X_{L1}=0.5$, $X_{L2}=0.93$, $X_T=0.15$
Electrical parameters	$P=0.9$, $Q=0.3$, $V_t=1.0<36^o$ $V_B=0.995<0^o$

III. Heffron-Philips Model

Heffron-Philips model for generator is linear model with 3 degrees in state space. Figure (2) shows this model with AVR loop.

The state space matrices corresponding to the presented model in Figure (2) can be formulated as equation (1).

Where $[x_1\ x_2\ x_3] = [\omega\ \delta\ e'_q]$ and $[u_1\ u_2] = [T_M\ \ u_E]$.

It is easy to calculate all the parameters of equation (1) by using table (1).

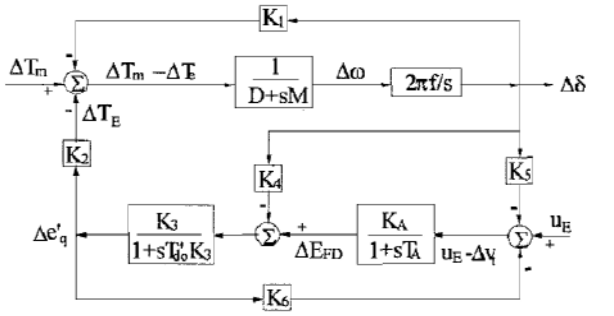

Figure 2. Heffron-Philips model with the AVR controller

$$
\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \\ \Delta \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ -\dfrac{k_1}{J} & -\dfrac{D}{J} & -\dfrac{k_2}{J} \\ -\dfrac{k_4}{k_3 T'_{do}} - \dfrac{k_5 k_A}{T'_{do}} & 0 & -\dfrac{1}{k_3 T'_{do}} - \dfrac{k_6 k_A}{T'_{do}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 & 0 \\ \dfrac{1}{J} & 0 \\ 0 & \dfrac{k_A}{T'ldo} \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} \qquad (1)
$$

IV. PSS CONTROL LOOP

Suppose a condition in which the second transmission line is removed under a fault [1]. The aim is to study rotor's speed oscillations and apply a pss to improve the stability resulting in decrement of speed deviations around the steady state value. Steady state values in such a condition can be calculated as equation (2).

$$
\begin{aligned}
V_{do} &= 0.6836, & V_{qo} &= 0.7298 \\
I_{do} &= 0.8432, & I_{qo} &= 0.4518 \\
\delta_o &= 79.13, & E_{fdo} &= 2.395
\end{aligned} \qquad (2)
$$

Using these values, coefficients k1 through k6 have below values.

$$
\begin{aligned}
k_1 &= 0.7643, & k_2 &= 0.8649 \\
k_3 &= 0.323, & k_4 &= 1.4187 \\
k_5 &= -0.12 & k_6 &= 0.3
\end{aligned} \qquad (3)
$$

Input signal of pss is speed of rotor and the output is a bounded voltage adding to the generator excitation. A block diagram for a regular pss is indicated in figure (3).

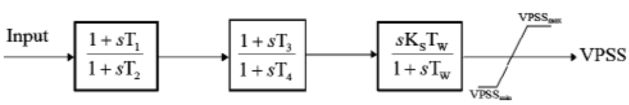

Figure 3. Block diagram of a regular pss

The response of speed deviations of the rotor after removing the second line with and without pss loop is shown in figure (4).

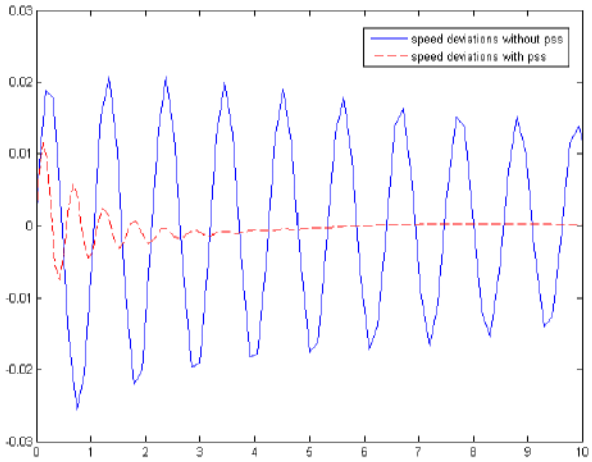

Figure 4. Speed deviations of rotor with and without presence of pss after the fault

Moreover, responses of power angle and deviations of terminal voltage of generator with and without pss can be seen in figure (5) and figure (6) respectively.

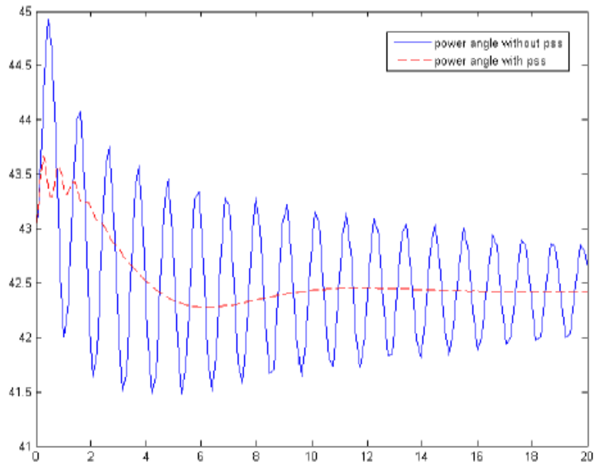

Figure 5. Power angle without and with presence of pss after the fault

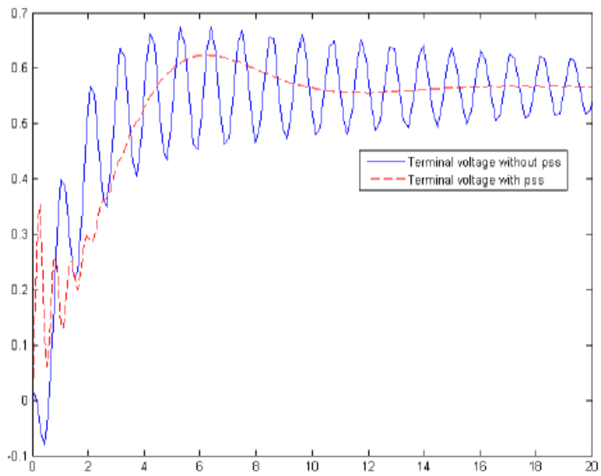

Figure 6. Terminal voltage of the generator with and without presence of pss after the fault

It is proved from figures (4), (5), and (6) that pss has been able to improve stability of the system actually by shifting the eigen values of the matrix A of state space equations to the left areas of s-plane. Since stability concept for a generator and related equipments is angular stability of a power system, you can say that pss extends the angular stability limits of a power system. However, pss can exacerbate instabilities of a power system. It is reasonable because pss is designed to operate around an operating point so it can improve the stability under small disturbances. However, when a large fault occurred, pss can lose synchronism of the generator by providing surplus excitation field. In addition, you may think of tuning a pss to extend more the angular stability but there is indeed a limitation to tune the parameters of pss since its output voltage reaches to saturation levels determined by the actual limitations. If the parameters of pss were selected in a way to have small amplitudes in state variables, the output voltage of pss becomes larger. On the other hand if they were designed in order to have small output voltage of pss, the amplitudes of state variables become larger. Therefore a compromise should be employed.

V. LQR BASED STATE FEEDBACK

Closed loop poles of a linear control system can be emplaced in desired places of s-plane using state feedback and observer control system design. Also its poles can be chosen by choosing appropriate observer gain. Response speed and estimation error dynamics can be defined by choosing closed loop poles. However, optimal selection of closed loop poles is really hard for industrial systems and real processes. Albeit an unstable system can be stabilized by applying states feedbacks, but linear optimal control systems should be engaged for below reasons.

The first reason is that it is hard to find appropriate closed loop poles in which the desired behavior of the system is satisfied. Selecting closed loop pole with great negative real parts makes the dynamic response of the system to be quick while the control effort to be greater than permissible levels. If selection of the closed loop poles makes saturation of control signals, dynamic behavior of the system will not be as same as the desired behavior even it may become unstable.

The second reason is noise which specially occurs in the systems with high gains.

Therefore optimal selection of closed loop poles will lead to a trade-off between speed of dynamic response and control effort.

Suppose the linear system in state space given by equation (4).

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y = Cx \tag{4}$$

The purpose is to stabilize the system so that all the state variables become zero by any initial value in maximum speed. There are many criteria to do that. Quadratic integral equation is a well-known such a criterion [2].

$$J = \frac{1}{2}\int_0^\infty (\underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u}) d_t \tag{5}$$

Where Q is a non-negative definite matrix which is called weighting matrix. On the other hand, R is a positive definite weighting matrix. Selection of values of these matrixes determines the dynamic speed of the controller as well as amplitudes of state variables and control signals. For example if R is selected small while Q is selected large, more stability will be attained with large control efforts.

Solving an optimal control problem means finding a u(t) by which equation (5) is minimized. Linear Quadratic Regulator, LQR, can solve this problem [2] (see equation (6)).

$$A^T X + XA - XBR^{-1}B^T X + Q = 0 \tag{6}$$

After calculating X from algebraic equation (6), k and u can be calculated from equations (7).

$$k = R^{-1}B^T X$$
$$u = -kx \tag{7}$$

Indeed, LQR is an optimal pole assignment and integral J defines circumstance of assigning closed loop poles as an optimizing criterion.

For our case study R and Q matrixes have been selected as equations (8).

It has been tried to have small control effort and equal importance to the state variables. It is important to note that there will be guarantied gain and phase margins by LQR. In fact it is not required to verify stability of the system after designing state feedback by LQR because it must be stable.

$$R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

VI. OBSERVER DESIGN

If A and C matrixes were observable in equation (4), then the observer can be designed as equation (9) [3].

$$\dot{\hat{x}} = A\hat{x} + Bu + L(C\hat{x} - y) \tag{9}$$

Where $\hat{x}$ denotes estimations of states x. $\overline{x} = \hat{x} - x$ is estimation error and it should reach to zero. $\overline{x}$ can satisfy equation (10).

$$\hat{x} = (A + LC)\overline{x} \tag{10}$$

Behavior of estimation error is related on the eigen values of matrix A+LC. For the mentioned case study L has been selected in a way to have -2, -5, and -11 as eigen values of A+LC (equation (11)).

L= [1696 -17.2 -2039.9] (11)

VII. LQR BASED POWER SYSTEM STABILIZER

The designed observer estimates all the three state variables. Then the estimated states are fed to the inputs by k which is designed using LQR.

Simulation results prove that state variables have significantly smaller amplitudes than those of pss while having small control signals too. Figure (7) indicates the speed deviations while figure (8) and figure (9) indicate the control signals. It can be found from figure (7) that LQR based pss has significant stabilizing characteristics in comparison with the traditional pss. Also figure (8) and figure (9) prove that control signals are much less in LQR based pss than the traditional one.

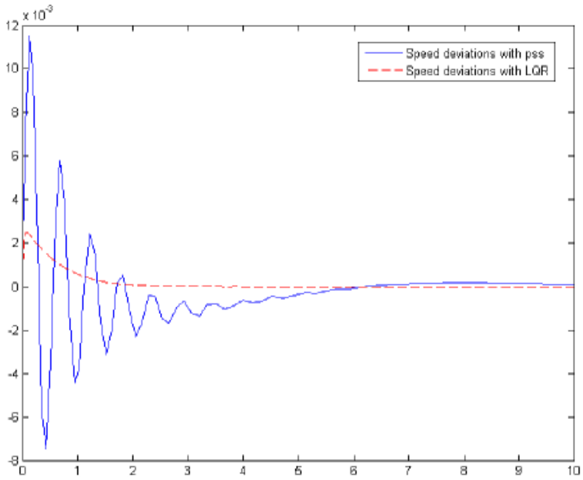

Figure 7. Comparison of speed deviation in LQR based pss and regular pss

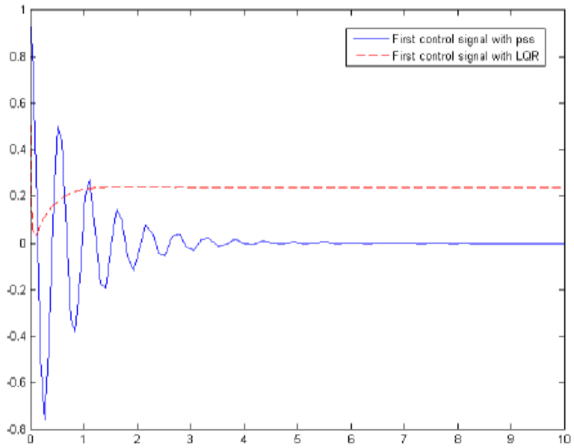

Figure 8. First control signal comparison between the two controllers

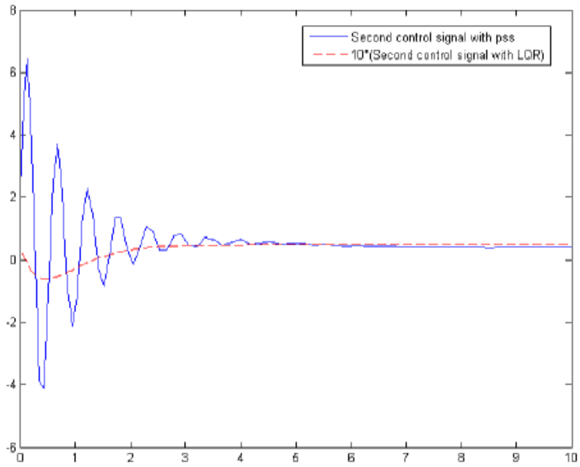

Figure 9. Second control signal comparison between the two controllers

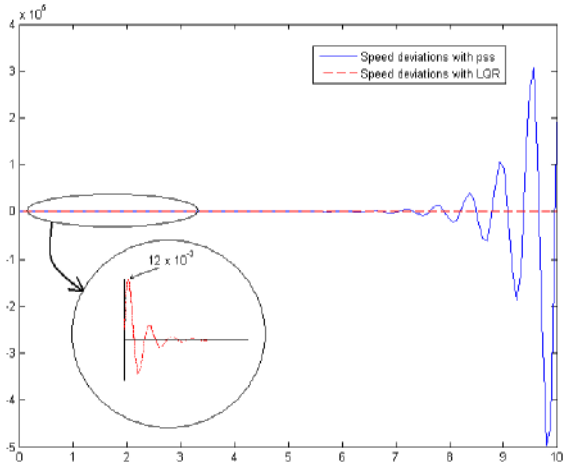

Figure 10. speed deviations of both controller after the heavy fault

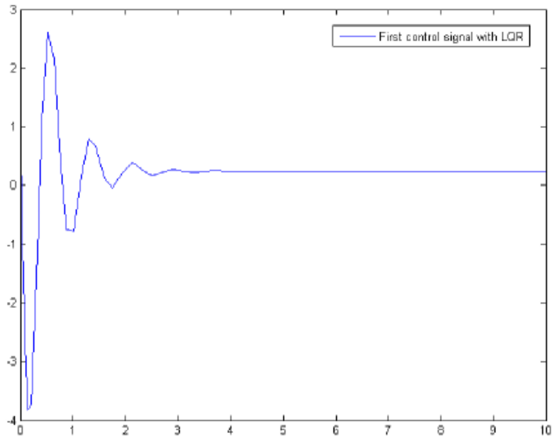

Figure 11. First control signal of the LQR base controller after the heavy fault

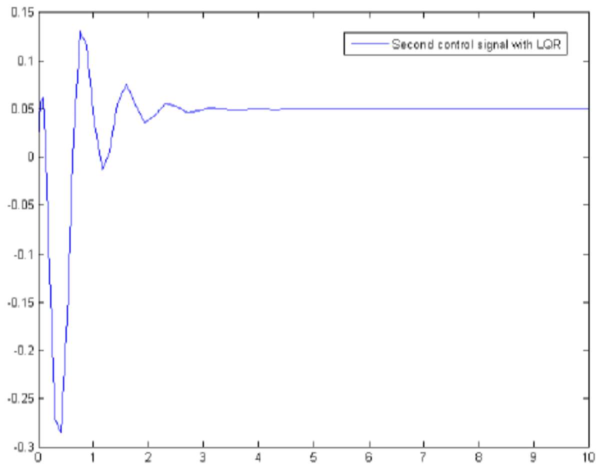

Figure 12. Second control signal of the LQR base controller after the heavy fault

Another simulation has been made to ensure that the LQR based pss has a good reliability. It has been supposed that a three phase short circuit at the received end of the second line has been occurred without removing the line. Figure (10) indicates speed variations of both controllers at the same condition. LQR based controller is still stable while traditional pss is unable to stabilize the power system. This result proves the extent gain and phase margins of the LQR solution. Figure (11) and figure (12) indicate that the LQR based controller is subjected to have greater control effort to remain stable.

VIII. Sliding Mode Based Power System Stabilizer

Reference [4] has proposed sliding mode to design a controller in order to have robust state covariance assignment. This method is briefly illustrated and is employed to have a robust controller for the stability problem of this paper.

The main aim of this controller is to reach the answer paths, trajectories, to a surface. Then the state variables should be tended to asymptotic stability through this surface. To achieve this aim the surface should be made attractive so that it can attract the paths. Hence the controller should be designed to tend the paths to the surface. The controller should have switching ability to keep the path by which the state variables have been reached to the desired surface. Different surfaces and control rules can be determined regarding the system model and control desires. This paper has engaged the proposed method in [4] to determine the plane.

Let plane S to be as (11).

$$S(t) = Cx(t) - \int_0^t (CA + CBG)x(\tau)d\tau \tag{11}$$

Where $S(t) = [S_1(t) \ldots S_i(t) \ldots S_m(t)]^T \in \Re^{m\times 1}$, C and G are constant matrices to be design, C is chosen such that CB is nonsingular, and G is the control feedback gain matrix which should be determined so that the state variables can fit the requirement in the sliding mode [4]. Equation (12) is attained by derivative of (11).

$$\dot{S}(t) = CBu - CBGx \tag{12}$$

Note that CD=0. In the sliding mode the states satisfy $\dot{S} = 0$, then we get the equivalent control as follows u_{eq} (t) =Gx(t). Equation (13) can be attained for dynamic sliding mode by replacing this input into system equation.

$$\dot{x} = (A + BG)x \tag{13}$$

If u(t) is anticipated as (14) in which $k > \|\Delta A\|$, α is a positive constant, $\operatorname{sgn}(S(t)) = [\operatorname{sgn}(S_1(t)) \ldots \operatorname{sgn}(S_m(t))]^T$, and $\|x\|$ denotes second norm of x, then the state of the system will converge to the sliding mode surface s(t)=0 with probability. We have chosen $C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \alpha = 1$ and k=1 so that CD=0 and CB is nonsingular. Eventually G is selected so that the eigen values of A+BG are {-3, -6± 5i}.

Let compare responses of certain models of LQR and sliding mode. Figure (13) represents the response of LQR based pss and Figure (14) is the response of sliding mode controller. It is proved from these figures that both controllers provide good stability condition for the power system however the amplitude of speed deviations in sliding mode is less than that of LQR controller.

Figures (15), and (16) present the speed deviations in LQR and sliding mode methods respectively under uncertain model. These results prove that the sliding mode controller is robust since even under uncertain model it has been successful to tend the state variables to the surface S. Moreover, Figures (17), and (18) denote the first and second control efforts of both controller.

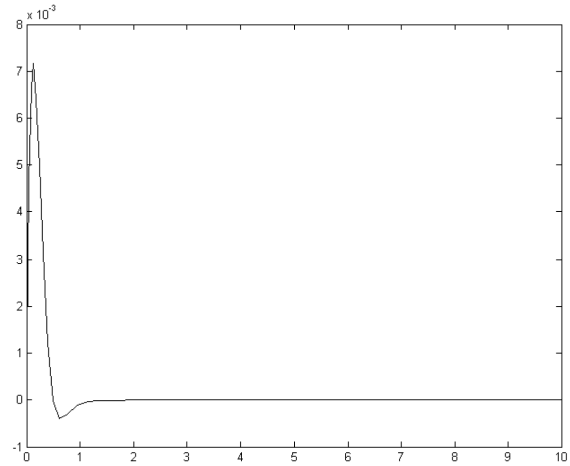

Figure 13. Speed deviations response of LGR based pss in certain model

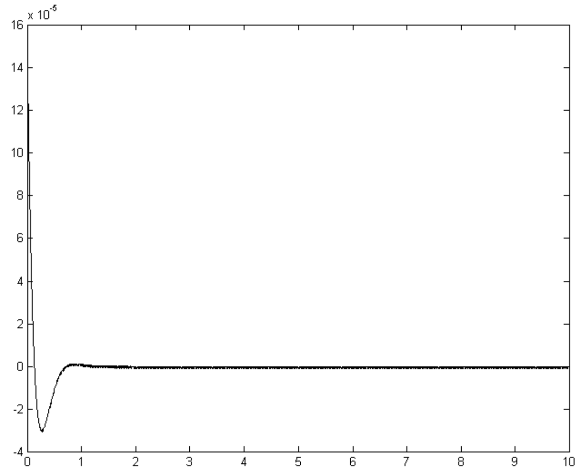

Figure 14. Speed deviations response of sliding mode based pss in certain model

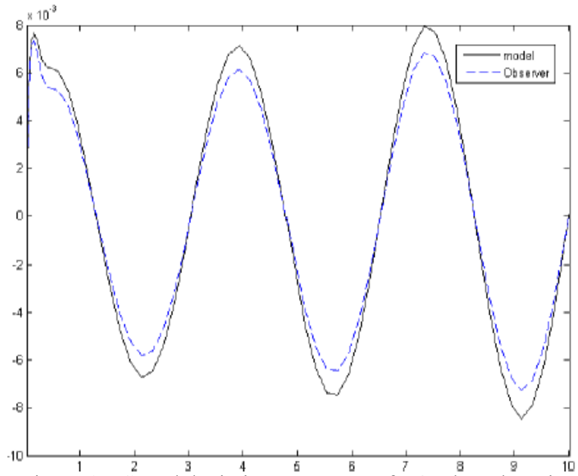

Figure 15. Speed deviations response of LQR based pss in uncertain model

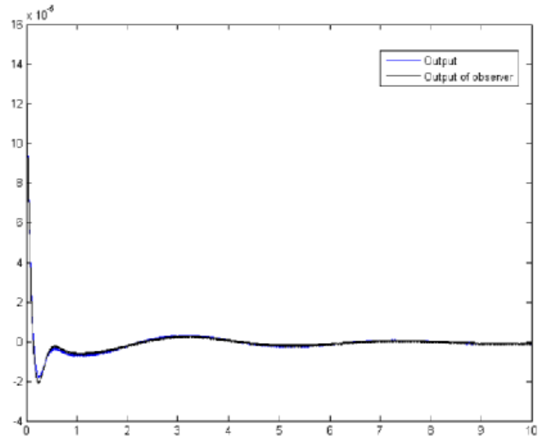

Figure 16. Speed deviations response of sliding mode based pss in uncertain model

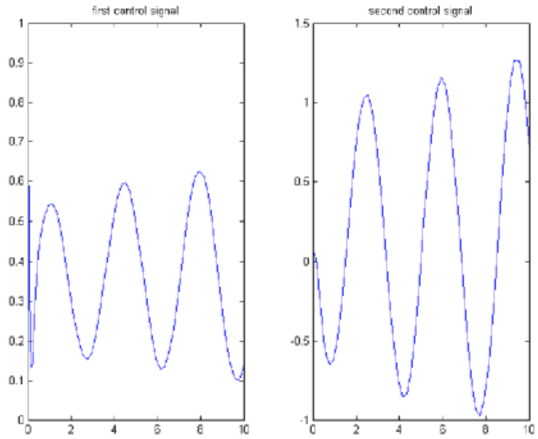

Figure 17. First and second control signals of LQR based pss in uncertain model

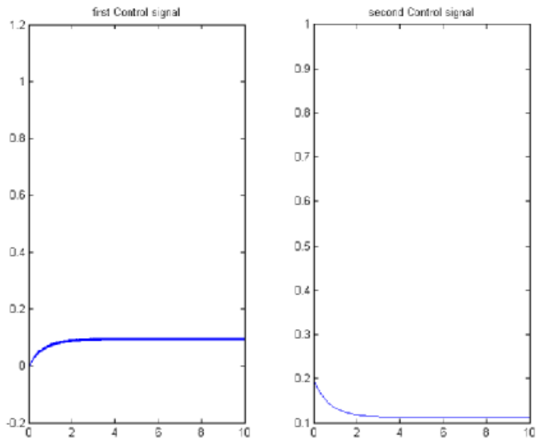

Figure 18. First and second control signals of sliding mode based pss in uncertain model

IX. Conclusion

Fundamental theory of regular pss, LQR based state feedback, observer, LQR based pss, and eventually sliding mode based pss were presented in the paper. Simulation results for a case study including regular pss and LQR based one were presented separately after removing a transmission line from service. LQR based pss yielded better stabilizing parameters while smaller control efforts than the regular pss. Moreover, after applying a heavy fault the regular pss got unstable while the LQR based pss was able to stabilize the power system although the control efforts were increased significantly. This phenomenon can prove the extent gain and phase margins of LQR based state feedback. Therefore a designer can apply LQR to design a power system stabilizer with good gain and phase margins without any worry about control signals since LQR can compromise between values of state variables and control efforts. At the last section, slide mode controller, which supposes to be robust, applied to the power system to pole placement. In certain model the results of both LQR and sliding mode were stable and acceptable taking it into consideration that the state variables have smaller magnitudes in the case of sliding mode controller. Under uncertainty, it was found that LQR controller which is not a robust controller became unstable while sliding mode controller was able to remain stable proving its robustness.

X. References

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