# Evaluation of Chaotic Ferroresonance in power transformers including Nonlinear Core Losses

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# Abstract

This paper investigates the effect of nonlinear core on the onset of chaotic ferroresonance and duration of transient chaos in a power transformer. The transformer chosen for investigation has a rating of 50 MVA, 635.1 kV, the data for which is given by Dommel etal. (Tutorial Course on Digital Simulation of Transients in Power Systems (Chapter 14), IISc, Bangalore, 1983, pp. 17\_38). The magnetization characteristic of the transformer is modeled by a singlevalue two-term polynomial. The core loss is modeled by a third order power series in voltage. With nonlinearities in core loss included, two effects are clear: (i) onset of chaos at larger values of open phase voltage, (ii) shorter duration of transient chaos, It also shown that nonlinear core can cause ferroresonance drop out.

#### 1. Introduction

Ferroresonance is initiated by improper switching operation, routine switching, or load shedding involving a high voltage transmission line. It can result in Unpredictable over voltages and high currents. The prerequisite for ferroresonance is a circuit containing iron core inductance and a capacitance. Such a circuit is characterized by simultaneous existence of several steady-state solutions for a given set of circuit parameters. The abrupt transition or jump from one steady state to another is triggered by a disturbance, switching action or a gradual change in values of a parameter. Typical cases of ferroresonance are reported in Refs. [1],[4]. Theory of nonlinear dynamics has been found to provide deeper insight into the phenomenon. Refs. [4],[7] are among the early investigations in applying theory of bifurcation and chaos to ferroresonance. The susceptibility of a ferroresonant circuit to a quasiperiodic and frequency locked oscillations are presented in Refs. [8],[ 9], the effect of initial conditions is investigated. Ref. [10] Isa milestone contribution highlighting the effect of transformer modeling on the predicted ferroresonance

oscillations. Using a linear model, authors of Ref. [11] have brought the effect of core loss in damping ferroresonance oscillations. The importance of treating core loss as a nonlinear function of voltage is highlighted in Ref. [12]. An algorithm for calculating core losses from no-load characteristics is given in Ref. [13]. The mitigating effect of transformer connected in parallel to a MOV arrester is illustrated in Ref. [14]. However, in all the references cited above, the effect of nonlinear core is either neglected. The present paper addresses the effect of nonlinear core on the global behavior of a ferroresonant circuit. The circuit under study represents a case of ferroresonance that occurred on 1100 kV system of Bonneville Power Administration as described in Ref.

### 2. Circuit Descriptions and Modeling

The three-phase diagram for the circuit is shown in Fig. 1.



The 1100 kV transmission line was energized through a bank of three single-phase as reported in autotransformers. Ref. [1] ferroresonance occurred in phase A when this phase was switched off on the low-voltage side of the autotransformer; phase C was not yet connected to the

transformer at that time. The autotransformer is modeled by a T-equivalent circuit with all impedances referred to the high voltage side. The magnetization branch is modeled by a nonlinear inductance in parallel with a nonlinear resistance and these represent the nonlinear saturation characteristic  $(\phi - i_{Lm})$  and nonlinear hysteresis and eddy current characteristics  $(v_m - i_{Rm})$ , respectively. The hysteresis and eddy current characteristics are calculated from the no-load characteristics by applying the algorithm given in Ref. [13]. The iron core saturation characteristic is given by:

$$\mathbf{i}_{\mathrm{Lm}} = \mathbf{s}_1 \boldsymbol{\phi} + \mathbf{s}_2 \boldsymbol{\phi}^q \quad (1)$$

The exponent q depends on the degree of saturation. It was found that for adequate representation of the saturation characteristics of a power transformer the exponent q may take the values 5, 7, and 11. In Ref. [15], the core loss is modeled by a switched resistor; which effectively reduced the core loss resistance by a factor of four at the time of onset of ferroresonance. In this paper, the core loss model adopted is described by a third order power series whose coefficients are fitted to match the hysteresis and eddy current nonlinear characteristics given in [1]:

$$i_{Rm} = h_0 + h_1 v_m + h_2 v_m^2 + h_3 v_m^3$$
 (2)

Per unit value of  $(i_{Rm})$  given in (3)

$$i(R) = -.000001 + .0047V - .0073V^{2} + .0039V^{3}$$



Fig 2.V-I characteristic of nonlinear core



The circuit in Fig. 1 can be reduced to a simple form by replacing the dotted part with the Thevenin equivalent circuit as shown in Fig. 4. By using the steady-state solution of MATLAB Simulink [16] with the data of the 1100 kV transmission line [1], Eth and Zth were found to be:







Fig. 5.Circuit of Ferro resonance investigations

# $E_{th} = 130.1 kv$ ; $z_{th} = -j1.01243E + 0.5\Omega$

The resulting circuit to be investigated is shown in Fig. 5 where Zth represents the Thevenin impedance. The behavior of this circuit can be described by the following system of nonlinear differential equations:

$$p2\phi = [e_{th} (t) - v_{c} - p\phi - R_{2} (s_{1} \phi + s_{2} \phi^{q} + h_{0} + h_{1} p\phi + h_{2} p\phi^{2} + h_{3} p\phi^{3})(4) - L_{2} (s_{1} p\phi + qs_{2} \phi^{(q-1)} p\phi)] [L_{2} (h_{1} + 2h_{2} p\phi + 3h_{3} p\phi^{2})]$$

$$\frac{p_{vc} = s_1 \phi + s_2 \phi^q + h_0 + h_1 p \phi + h_2 p \phi^2 + h_3 p \phi^3}{C}$$
(5)

With these parameters:

$$\begin{split} V_{base} &= 635.1 Kv, \ I_{base} = 78.72 A, \ R_{base} = 8067 \Omega, \\ L_{S\ p.u} &= .0188, C_{p.u} = .07955, R_{p.u} = .0014 \ Rcore = 556.68 \ p.u \end{split}$$

The formulation with  $\phi$ ,  $p\phi$  and  $v_c$  taken as state variables are given by:

$$x_{1} = \phi ; x_{2} = p\phi ; x_{3} = v_{c} \quad (6)$$

$$px_{1} = x_{2} \quad (7)$$

$$px_{2} = [e_{th} (t) - x_{3} - x_{2} - R_{2} (s_{1} x_{1} + s_{2} x_{1}^{q} + h_{0} + h_{1} x_{2} + h_{2} x_{2}^{2} + h_{3} x_{2}^{3}) - L_{2} (s_{1} x_{2} + q s_{2} x_{1} \phi^{(q-1)} x_{2})]$$

$$(B)$$

$$\frac{px_{3} = s_{1} x_{1} + s_{2} x_{1}^{q} + h_{0} + h_{1} x_{2} + h_{2} x_{2}^{2} + h_{3} x_{2}^{3})}{C}$$
(9)

For these values of q, system simulated:

for q = 11  $s_1 = .0667$ ,  $s_2 = .0001$ for q = 7  $s_1 = .0067$ ,  $s_2 = .001$ for q = 5  $s_1 = .0071$   $s_2 = .0034$ 

Equations of system with linear core are:

$$E - V_{c} - R_{1}\left(\frac{d\lambda}{dt} + a\lambda + b\lambda^{q}\right)$$
$$- L_{1}\left(\frac{d^{2}\lambda}{dt^{2}} + a\frac{d\lambda}{dt} + bq\lambda^{q-1}d\lambda\right) - d\lambda = 0$$
$$\dot{V}_{c} = \frac{1}{c}\left(\frac{d\lambda}{dt} + a\lambda + b\lambda^{q}\right)$$



Fig 6.Phase plan diagram for q=5 with linear Core loss



Fig 7. Phase plan diagram for q=7 with linear Core loss



Fig 8.phase plane diagram for q=11 with linear Core loss

### 3. Simulation Results and Discussion

Time domain simulations were performed using fourth order Runge\_Kutta method and validated against Matlab Simulink. The initial conditions as calculated from steadystate solution of Matlab are:

 $x_1 = 0.0$ ;  $x_2 = 1.67$  pu;  $x_3 = 1.55$  pu)

The circuit in Fig.5 is analyzed by first modeling the core loss as a constant linear resistance. Figs. 6, 7and Fig. 8 show the phase plan diagram for q=5, 7 and q=11 with nonlinear core. Bifurcation diagram for q= 5, 7 and 11 generated by conventional time domain simulation show in Figs.12, 13 and 14. It was found that the chaotic behavior begins at a value of  $(E_p = 0.8 pu)$  for q= 5 and  $(E_p = 1 pu)$  for q=7,11 where represents the amplitude of eth (t).Transient chaos settling down to the source frequency periodic solution was observed for some values of parameters as shown in Fig. 12 for a typical value of  $E_n$  in the chaotic Region. Conventional bifurcation diagrams with the nonlinear model of core loss are presented in Figs. 15, 16 and Fig. 17. Figs.9, 10 and 11 illustrates the corresponding phase plot from which a period 9 oscillation can be discerned for q=11.



Fig10. Phase plane diagram for q=7 Nonlinear core loss







Bifurcation Diagram for q=7 With Linear Rcore



Fig. 13. Bifurcation diagram for q=7 with linear core loss









Fig.16. Bifurcation diagram for q=7 with nonlinear core loss



Fig.17. Bifurcation diagram for q=11 with nonlinear core loss

# 4. Conclusion

The dynamic behavior of a transformer is characterized by multiple solutions. Inclusion of nonlinearity in the core loss reveals that solutions are optimistic when compared with linear models. An operational guideline for the range of values of the core loss that avoids the jump between steadystate solutions and contains over voltages, is provided.

# **Appendix: Nomenclature**

- *a*, *b*, *c* index of phase sequence
- h0, h1, h2, h3, h4 coefficient for core loss nonlinear function
- *n* index for the neutral connection
- *s1* coefficient for linear part of magnetizing curve
- *s2* coefficient for nonlinear part of magnetizing curve
- *q* Index of nonlinearity of the magnetizing curve
- *Zth* Thevenin's equivalent impedance
- *C* linear capacitor
- *Rm* core loss resistance
- *L* nonlinear magnetizing inductance of the transformer
- *i* instantaneous value of branch current

v instantaneous value of the voltage across a branch element

eth(t) instantaneous value of Thevenin voltage source

*e* instantaneous value of driving source

- *p* time derivative operator
- *Eth* R.M.S. value of the Thevenin voltage source
- *Ep* peak value of the Thevenin voltage source
- *x* state variable
- f flux linkage in the nonlinear inductance
- *v* angular frequency of the driving force

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