

# Implementation of Inter-area Angle Stability Prediction in Wide Area Control

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**Abstract:** This paper proposes a new interarea angle stability prediction algorithm. This proposed algorithm does not require any prior knowledge of system state as it operates directly from measurements drawn from PMUs. The proposed predictor foresees the system stability state for 500 ms in advance. Applying the COI angle concept for the interconnected power systems, inter-area stability can be predicted in a proper time. The proposed method was applied for two standard test systems, two area 4 generator system and IEEE 50 generators test system. The results showed that the prediction accuracy in most cases converges around 1%. At the worst case the predicted values approach 7%.

## 1. INTRODUCTION

Power systems are large interconnected nonlinear systems. They respond to a disturbance over a varying timescale, ranging from milliseconds even to hours. Wide range of contingencies and disturbances occur and abnormal operating conditions may be detected. A different corrective control schemes must be implemented in each type of contingency or disturbance at the right time [1].

Because of deregulation, many power systems around the world are being forced to operate closer to their stability limit because of the operational requirements in an open access environment and the environmental considerations. Power transfers across the interconnected areas of a power system are unpredictable due to market price variations. Inter-connection unforeseen operating conditions and area instability can lead to system blackout [2].

The recent series of blackouts in different countries has further emphasized the need for operators to have better information regarding the actual state of the power system they are operating.

For the last two decades, advances in computer technology and communications provide the operators with the information needed for appropriate control action. At the same time, measurement systems based on phasor measurement units (PMUs) are becoming proven technology and are seen by many utilities as one of the most promising ways of providing phasor information for wide area control [3].

Since many real-time operating decisions, both manual and automatic, are based on software applications using information derived from the phasor measurements through communication, these developments have shown immediate benefits in terms of increased accuracy, stability, and speed of convergence of control action decision making.

Current software and algorithms used in wide-area control are based on phasor measurements of bus voltage and generator reactive power [1]. In some applications, it is effective to use phase angle measurements to detect inter-area angle instability [1].

## 2. PROBLEM FORMULATION

If a severe disturbance, such as three-phase fault occurs in a tie-line between two areas of an interconnected power system, some of the generators of the power system may accelerate and may lose synchronism. If such a disturbance is not cleared at a proper time, the loss of synchronism may extend to other areas' generators and a blackout may occur [4].

The loss of synchronism after fault clearing is affected by the increase or decrease of the generator relative rotor angle beyond an identifiable threshold.

Phase angle prediction can be used to detect the first swing instability in advance, which gives time to operator to apply the proper preventive control action. Based on phase angle prediction and frequency measurements of critical generators buses from the entire interconnected power system transient stability can be detected and mitigated.

## 3. PROPOSED ALGORITHM

The phase angle of generator bus and relative phase difference varies through a wide range during system operation. The concept of center of inertia (COI) for the computation of the system phase angle reference is used to determine the interconnection phase angle [3]. This approach is used to quantify the extent of phase angle variations away from the system center.

Since the internal rotor angle cannot directly measured, we approximate the internal angle with the phase angle of generator bus which is normally monitored by PMU [3].

The proposed algorithm is divided into two parts, the first part is calculating the system COI through data collection from PMUs placed on generators bus. The second part, a fast learning algorithm is used to predict

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the fore coming values for COI and consequently determine system stability measure.

### 3.1 Center of Inertia COI

The COI is calculated in [5] as follows:

$$\delta_{COI} = \frac{\sum_{i=1}^N \bar{\delta}_i H_i}{\sum_{i=1}^N H_i} \quad (1)$$

Where  $\bar{\delta}_i$  is the internal generator rotor angle,  $H_i$  is the respective generator inertia time constant and  $N$  is the total number of generators of the system areas.

As the machine inertia is directly proportional to the real power output, the weight factor of generator inertia time constant can be replaced by generator real power  $P$ .

For phase angle measurements in an area  $i$  in interconnected power system, we have the approximate center of inertia COI angle reference:

$$\delta_{COI}^i = \frac{\sum_{j=1}^N \delta_j^i P_j^i}{\sum_{j=1}^N P_j^i} \quad (2)$$

Where  $P_j^i$  denotes the real power generation schedule at generating plant  $j=1, \dots, N$  in area  $i$ .

Increasing the number of phase angle measurements, in each area of interconnected power system, accuracy of  $\delta_{COI}^i$  computation can be increased.

For the entire interconnected power system, the overall center of inertia  $\delta_c$  can be calculated as follows:

$$\delta_c = \frac{\sum_{i=1}^N \delta_{COI}^i P^i}{\sum_{i=1}^N P^i} \quad (3)$$

### 3.2 Prediction Algorithm

The proposed prediction algorithm is an adaptation of a proven robotic ball-catching algorithm and has been applied to power system instability prediction as in [4].

The prediction algorithm is divided into two parts described in [6]:

The coarse tuning which is a tracking stage of interconnection center of inertia,  $\delta_c$ , and

The fine-tuning which is the extended prediction of center of inertia,  $\delta_c$ .

#### A. Coarse tuning

The actual calculated center of inertia angular position is represented by  $\delta_c$  and the predictor is presented by  $\varphi_p$ . The algorithm requires the projection

of angular position  $\delta_c$  in x-y coordinates and so as to the predictor position.

The x-y components of the predictor will be:

$$\begin{aligned} \varphi_{px} &= \cos(\varphi_p) \\ \varphi_{py} &= \sin(\varphi_p) \end{aligned} \quad (4)$$

and the center of inertia angular position will be represented by:

$$\begin{aligned} \delta_{cx} &= \cos(\delta_c) \\ \delta_{cy} &= \sin(\delta_c) \end{aligned} \quad (5)$$

For each sampling instant of PMU the center of inertia angular position x and y components will be updated.

Equations 4, and 5 will be substituted into the coarse tuning objective function:

$$\text{Min}_{T_f, \varphi} \left\{ T_f - \alpha(\delta_c \cdot \varphi_p) + \beta[(\delta_{cx} - \varphi_{px})^2 + (\delta_{cy} - \varphi_{py})^2] \right\} \quad (6)$$

where,  $T_f$  is a rough estimate of the final prediction time obtained from the equation:

$$\varphi_p - \delta_c + T_f (\dot{\varphi}_p - \dot{\delta}_c) = 0 \quad (7)$$

$\dot{\delta}_c$ ,  $\dot{\varphi}_p$  are the center of inertia and predictor velocities respectively, and are represented by:

$$\begin{aligned} \dot{\delta}_c &= \omega_c = \frac{d\delta_c}{dt}, \\ \dot{\varphi}_p &= v_p = \frac{d\varphi_p}{dt} \end{aligned} \quad (8)$$

and  $\alpha$  and  $\beta$  are weighting functions.

The minimization of the objective function is obtained by differentiation of equation (6). The predicted center of inertia angular velocity  $v_p$  in y-direction is driven through the following equation:

$$A_1 v_{py}^3 + A_2 v_{py}^2 + A_3 v_{py} + A_4 = 0 \quad (9)$$

where,  $A_1, A_2, A_3$ , and  $A_4$  are functions of  $\delta_{cx}$ ,  $\delta_{cy}$ ,  $\omega_{cx}$ ,  $\omega_{cy}$ ,  $\alpha$ , and  $\beta$  which are known.

The predicted center of inertia angular velocity  $v_p$  in x-direction is driven through manipulating equations (6) and (7)

$$v_{px} = \omega_{cx} + \frac{(\varphi_{px} - \delta_{cx})}{(\varphi_{py} - \delta_{cy})} (v_{py} - \omega_{cy}) \quad (10)$$

The optimum predicted center of inertia angular velocity vector can then be obtained, through Solving equation (9) for  $v_{py}$  and equation (10) for  $v_{px}$ .

For each sampling instant the difference between the measured center of inertia  $\delta_c$  and the predicted center

of inertia  $\varphi_p$  is calculated and compared to a predetermined tolerance  $TOL$ . If the difference between the actual and predicted center of inertia is less or equal than pre-specified tolerance [ $TOL \geq (\varphi_p - \delta_c)$ ], the algorithm is switching into the fine-tuning stage of prediction.

### B. Fine tuning

The purpose of this stage is fine tune the predicted center of inertia value  $\varphi_p$  after a period of  $T_f$  regarding the  $\varphi_p$  trajectory generated from the coarse tuning process. Taylor series expansion has been proven as a good estimator for unknown data. It is used in this stage to fine tune the values obtained from the previous coarse tuning stage.

Angular center of inertia velocity varies continuously and shows a smooth change because of the large inertia of turbine-generator combination of interconnected area power system. In order to fine tune the predicted angular velocity  $\nu_p$ , and center of inertia angle  $\varphi_p$ , a function extrapolating the three measured points  $\omega_c(\zeta_0)$ ,  $\omega_c(\zeta_1)$ , and  $\omega_c(\zeta_2)$  is defined using Taylor series expansion given by:

$$\nu_p(T_f) = \omega_c(\zeta_2) + \kappa_1(T_f - \zeta_2) + \kappa_2(T_f - \zeta_1)(T_f - \zeta_2) \quad (11)$$

where;  $\zeta_0$ ,  $\zeta_1$  and  $\zeta_2$  are the last three time stamps for PMU measurements.

$T_f$  = the prediction period in sec.

$$\kappa_0 = (\omega_c(\zeta_1) - \omega_c(\zeta_0)) / (\zeta_1 - \zeta_0)$$

$$\kappa_1 = (\omega_c(\zeta_2) - \omega_c(\zeta_1)) / (\zeta_2 - \zeta_1)$$

$$\kappa_2 = (\kappa_1 - \kappa_0) / (\zeta_2 - \zeta_0) \quad (12)$$

Integrating equation (11), the function predicting the center of inertia  $\varphi_p$  will be given by:

$$\varphi_p(T_f) = \int_{\zeta_0}^{T_f} \nu_p(t) + \delta_c(\zeta_0) \quad (13)$$

By using the algorithm above, area said to be losing synchronism can be predicted without any prior system configuration and exhausting system calculations. It will be shown by application that the proposed system can predict inter foreseen area instability in a time window permit the application of proper preventive control with acceptable prediction error.

## 4. APPLICATION TO INTERCONNECTED POWER SYSTEM

### 4.1 Simple Test System

The applicability of the proposed algorithm to a wide area power system is first studied with a simple system of two areas. The basic topology is depicted in Fig. 1 [5].

The system contains eleven buses and two areas, connected by a weak tie between buses 7 and 9. The left half of the system is identified as area 1 while the right half is identified as area 2.

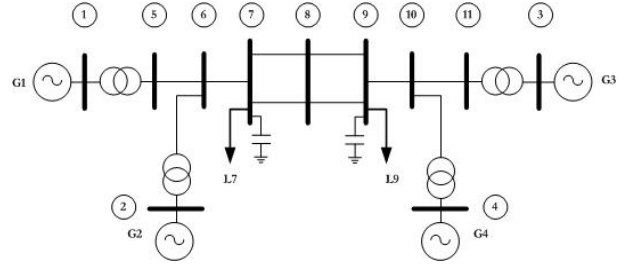


Fig. 1 Single line diagram of simple test system.

#### 4.1.1 Stable conditions

##### A. Generators COI angles prediction

The system is simulated using Matlab software. The test system contains eleven buses, four generators, four transformers, two shunt capacitors, and two loads. The technical data of the system are obtained from [5].

A three phase fault is initiated at 0.5 s at bus nine of the studied system. The duration of the fault is 0.1 s and is cleared by opening the line 9-8 at 0.6 s. Simulation results showed that the system is stable.

The application of the prediction algorithm showed that the COI generator angles can be predicted for 500 ms with an acceptable accuracy.

Figure 2 shows the actual and predicted COI angles for the four system generators. It is obvious that the predicted values are very close to the actual COI measured values.

The prediction errors between the actual and predicted COI of the four generator buses 1, 2, 3, and 4 are also calculated. Figure 3 shows the prediction error of the four COI and the error is very small compared to the actual value of COI angles.

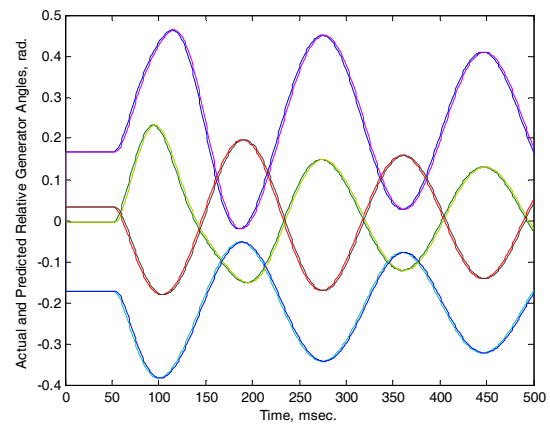


Fig. 2 Actual and predicted COI angles for a prediction time of 500 ms.

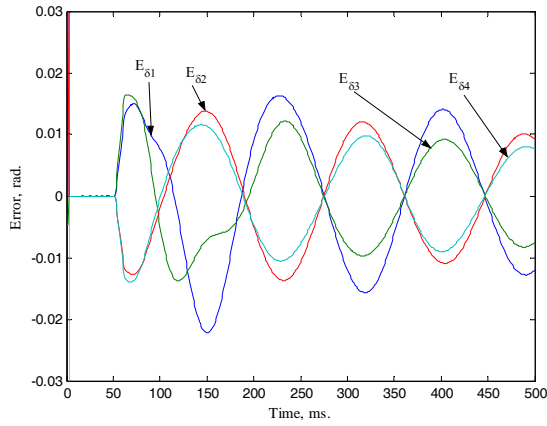


Fig. 3 Prediction error for COI bus angles.

### B. Two area systems:

The previously 4 generators test system is divided into two area as given in [5]. Area 1 contains buses 1, 2, 5, 6, and 7, while area A2 contains buses 3, 4, 9, 10, and 11. The two areas are connected through a tie line between bus 7 and 9. The same fault and clearing conditions are applied to the two areas system. The proposed algorithm is applied to the test system to predict the two areas COI angles. Figure 4 shows the simulation result for 500 ms. Results verify that the COI angles of the two areas can be predicted with an acceptable accuracy. From figure 5, the prediction error for the COI angles is around 0.01 radian through most of the prediction period.

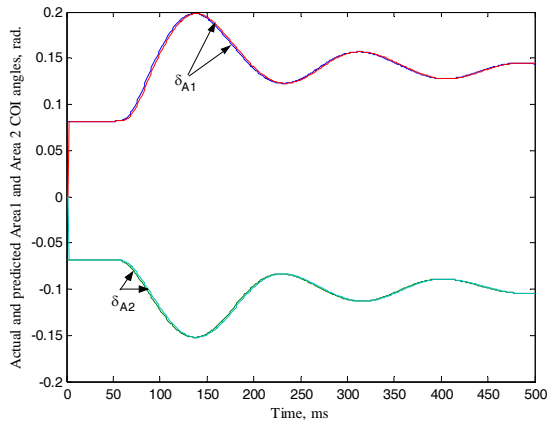


Fig. 4 Actual and predicted COI angles for area 1 and area 2

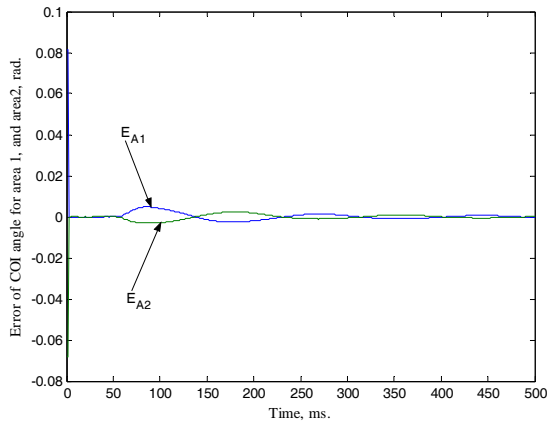


Fig. 5 Error between actual and predicted COI angle for area 1 and area 2

### 4.1.2 Unstable conditions

When applying a three phase fault at bus 6 at a fault time 0.5 s and clearing the fault 0.11 s later removing line 6-7 at 0.61 s, the system gone unstable. The proposed algorithm is applied to the unstable condition and the prediction results are given in figure 6. Area A1 COI angle decreases in an exponential manner to reach a value of -22.5 radians.

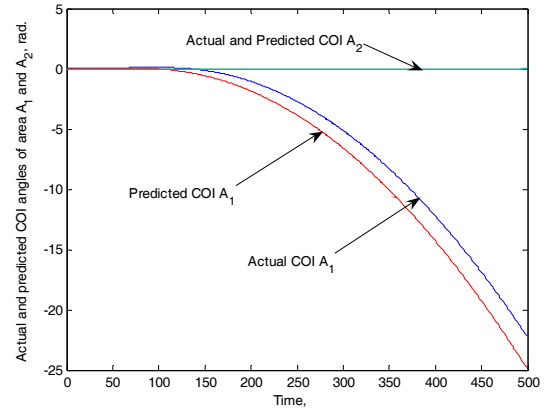


Fig. 6 Actual and Predicted COI angle for two areas network.

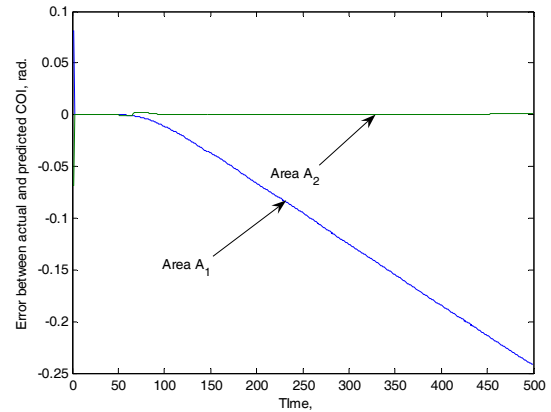


Fig. 7 Error between actual and predicted center of inertia of two areas system

Figure 7 shows the error between actual and predicted COI angles and it was found that the error value increases for unstable area A1 to reach about -0.24 radian.

### 4.2 IEEE 50 Gen-145 bus Test System

The investigation is, further extended to a larger test system. The IEEE 50 generator test system given by [7] is considered. The system data are given in table 1. Full system details can be found in [7].

Table 1: IEEE 50 Generator Test System Data

No. of Generators	50
No. of Buses	145
No. of Transmission Lines	401
No. of Transformers	52
No. of PV Buses	49
No. of Loads	64
No. of Shunt Reactance	97
System Frequency	50 Hz
Slack Bus	100

To investigate the effectiveness of the proposed algorithm, a three-phase fault was applied at bus 7. Fault was initiated at 0.1 second and cleared at 0.208 second by tripping line 6-7. This fault created an unstable condition for the system as generator at bus 104 (Generator #2) accelerating increasingly as shown in figure 8. In the meanwhile, the other 49 generator maintain equilibrium.

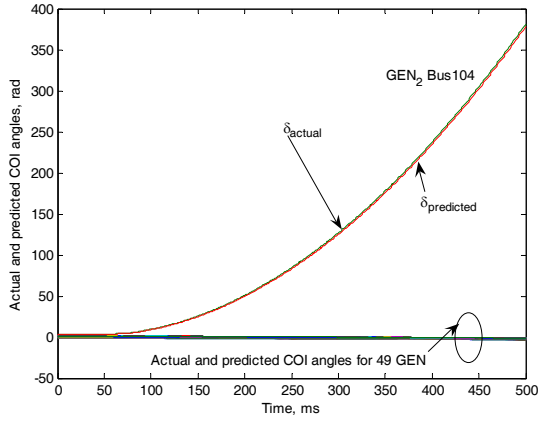


Fig. 8 Actual and predicted generator angle referred to COI at fault conditions

The prediction error is calculated and plotted to show the deviation from the actual COI angle. Figure 9 shows that the prediction error of generator 2 at bus 104 is increasing till a maximum value of 3.25 radians.

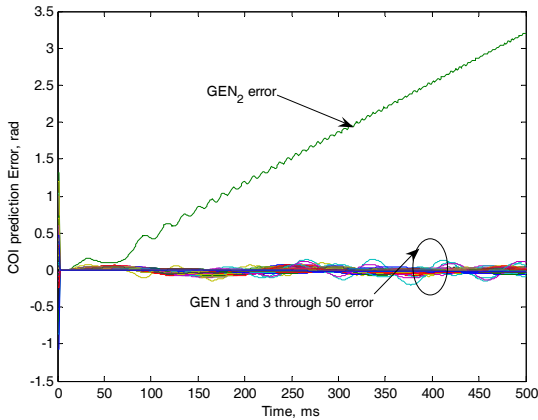


Fig. 9 Prediction error in radians

Separating generators (1, and 3 through 50) prediction error; figure 10 shows that prediction error clustered around zero with maximum value of  $\pm 0.08$  radians.

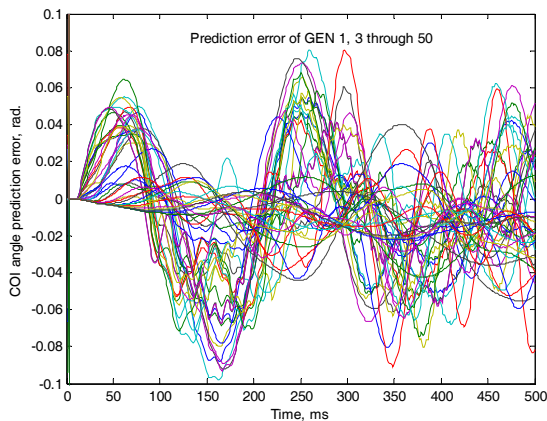


Fig. 10 Prediction error of stable 49 generators

Considering the wide area control and the need to divide the network into interconnected control areas; the system is divided into two areas as shown in table 2[7].

Table 2: IEEE 50 Generator Areas

Area No.	Generator Bus No.
1	115, 116, 130, 131, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145
2	60, 67, 79, 80, 82, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 117, 118, 119, 121, 122, 124, 128

Figure 11 shows the COI angle of the two area IEEE-50 generator system. For the fault conditions described above area 2 COI accelerate and the area is going to an unstable zone. Area 1 is stable as the variation of the inter area COI angle is approximately steady.

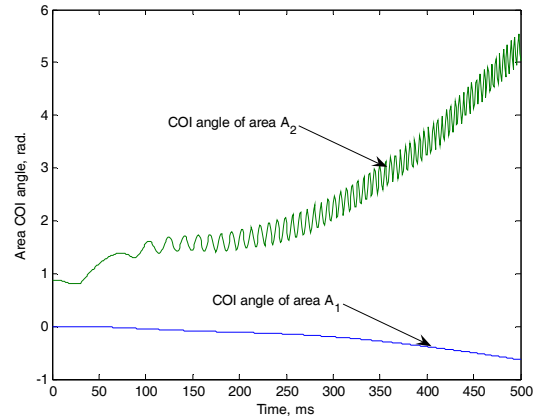


Fig. 11 The distribution of two areas COI angle along simulation time.

Predicting the two areas COI angles, the results shown that a precise approach to the values which have been calculated before.

Figure 12 shows the actual and predicted COI angles of the IEEE-50 generators two control areas. It is clear that the predicted values converged too close to the actual value.

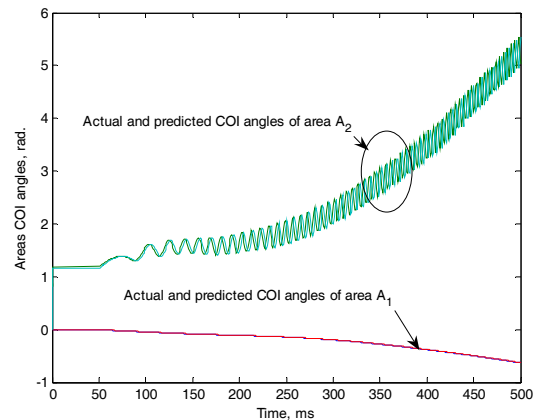


Fig. 12 Actual and predicted COI angles for 500 ms prediction period.

The accuracy of the predicted values is explained calculating the error between the predicted and the

actual COI angles. Figure 13 shows that the prediction error does not exceed 0.3 radians.

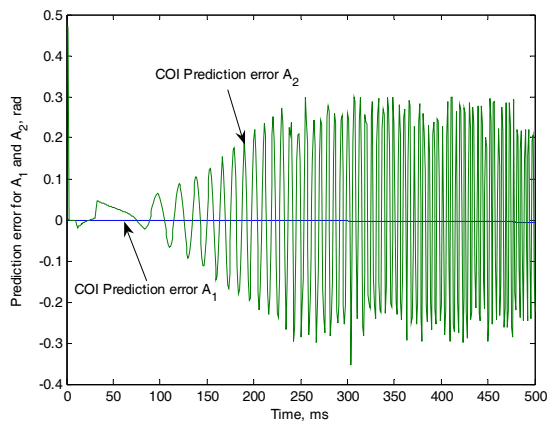


Fig. 13 Prediction error for 500 ms prediction period.

## 5. CONCLUSIONS

The proposed algorithm is adopted from a proven robot control system and the technology of PMU [2, 3, 6]. The algorithm shows a good accuracy regarding the actual COI angle prediction. No prior knowledge of system state is required. PMU measurements can predict the system stability state for 500 ms in advance. Wide area monitoring and control can be achieved on-line using the proposed algorithm. Severe contingencies can be prevented at the right time as foreseen system state can be predicted in advance. Further investigation on wide area control, such as generator tripping and load shedding, using the proposed method is in progress to proof the applicability of the algorithm.

## 6. REFERENCES

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