# Reactive power in one-phase circuits with periodical voltages – reactive power compensation

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***Abstract* - The article states about reactive power compensation methods for circuits with non-sinusoidal voltages, there have been presented selected theories application in order to compensate the reactive power in one-phase circuits. Also measurement results after compensation of an actual object supplied from an non-sinusoidal voltage source were presented. Also algorithms of optimal capacity selection were given, which connected in parallel to the circuit with inductive character will cause current root-mean-square value minimization.**

## I. Introduction

Significant part of electrical energy receivers draw active energy and convert it for work and heat, as well as reactive energy. Reactive power describes the process of electrical and magnetic field energy exchange. The process of energy exchange can proceed independently from work and heat exchange process. The utilization measure of energy supplied to receiver is the power factor:

$$PF = \frac{P}{S} \tag{1}$$

where: PF – power factor, P – active power, S – complex power.

As complex power we name the biggest active power value, which can occur for given root-mean-square value of voltage and current:

$$S = UI \tag{2}$$

where: $U_{RMS}$ – voltage root-mean-square value, $I_{RMS}$ – current root-mean-square value.

If the receiver works with small power factor, it means that it draws bigger current than it is necessary. If the reactive power is drawn from distant sources it causes a rise of supply currents and in consequence rises the transmission losses. As a result it is needed to increase the transmission line conductor cross section and lowering the ability to load the generators and transformers with active power. All this aspects are causing increase of operation costs incurred by the electrical energy recipients.

## II. Reactive power in circuits with periodic voltages

Reactive power is correctly defined for linear circuits with sinusoidal voltages (3):

$$Q = UI \sin\varphi \tag{3}$$

Basing on above mentioned dependence (3) it is possible to select the optimal capacity with which we will achieve power factor correction:

$$C = \frac{Q \cdot T}{U^2 \cdot 2 \cdot \pi} \tag{4}$$

Whereas for non-linear circuits or circuits supplied with distorted voltages many power theories exist. In this article selected reactive power theories will be demonstrated and used to compensate an actual object (choke with ferromagnetic core) supplied from public network.

## III. Reactive power in circuits with non-sinusoidal voltages

Most disseminated reactive power theory for non-sinusoidal voltages and currents is the one defined in 1927 by C.I. Budeanu [1]:

$$Q = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n \tag{5}$$

where: n – harmonic number, $\varphi_n$ - phase displacement between voltage and current of the n-th harmonic.

From the beginning the reactive power theory proposed by Budeanu, besides many adherents, had many opponents. The main charges for this theory are lack of physical deformation power interpretation and unauthorized oscillatory components summation of particular harmonics. In 1987 L.S. Czarnecki has published a scientific work in which he criticized the Budeanu power theory [2]. He advanced a hypothesis about its worthlessness for the sake of lack of possibilities to:

- minimize the reactive power and therefore no possibility to correct the power factor,
- reactive power $Q_B$ is no measure of energy oscillation,
- it doesn't enable the calculation of capacitor capacity, by which the power factor is the biggest,
- it suggests incorrect energetic phenomena interpretation with distorted voltages.

One of the advantages arguing for this theory usage is its conservatism and relatively simple measuring devices construction. However this argument, in actual signal processing theory state, is insufficient according to the article Author.

In the twenties of 20th century M. Iliovici has presented a reactive power interpretation as loop area which is made by current and voltage coordinates [5].

$$Q = -\frac{1}{2\pi}\oint idu \qquad (6)$$

Characteristics in I, U coordinates of non-linear objects are usually complex and create multiple loops and furthermore their shape changes strongly under the influence of voltage change. Areas inside loops are circulated clockwise or counterclockwise. Therefore the energy of electric or magnetic field is sometimes drawn and sometimes returned. We select the capacitor capacitance connected to the object to achieve a state in which the resultant loop area will be equal zero, and so the reactive power will be equal zero. But it doesn't mean that magnetic and electric field energy won't be mentioned. If energy is not mentioned the object characteristic in I, U coordinates is reduced to a line segment.
The reactive power compensation basing on above mentioned theories is made using the formula (7), for given supplying voltage we can determine an optimal capacity, which connected to circuit (Figure 1) will cause rms value decrease.

$$C_{opt} = \frac{2\cdot\pi\cdot Q}{\dot{u}^2\cdot T} \qquad (7)$$

where: Q – reactive power, $\dot{u}$ - voltage derivative rms value, T – time period.

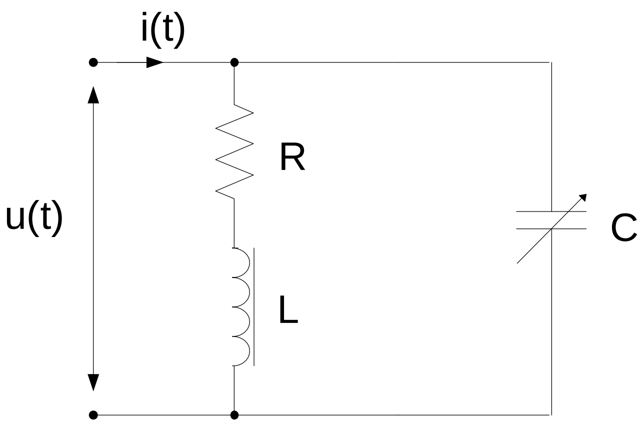


Figure 1. Parallel compensation.

In 1931 S. Fryze proposed reactive power determination for distorted voltages in such a way that the power equation is fulfilled (8) [4], [5]:

$$S = \sqrt{P^2 + Q^2} \qquad (8)$$

Fryze thought that power is a too elementary concept to define it with assistance of such complicated instrument like Fourier series, which was introduced by Budeanu. In his theory Fryze distributed current to sum of two, mutually orthogonal, currents:

$$i(t) = i_a(t) + i_b(t) \qquad (9)$$

where: $i_a(t)$ – active current component, $i_b(t)$ – reactive current component.

And so the reactive power was equal (10):

$$Q_F = |U||I_b| \qquad (10)$$

where:

$$i_a(t) = G_e u(t) \qquad (11)$$

Where Ge can be determined from (12):

$$G_e = \frac{P}{\|u\|_{L^2}^2} = \frac{\frac{1}{T}\int_0^T u(t)i(t)dt}{\frac{1}{T}\int_0^T u^2(t)dt} \qquad (12)$$

An unquestionable advantage of Fryze's theory is elimination, form initial Budeanu theory, of fourier series and third power component (deformation power).

For many years there hasn't existed a generally accepted theory describing energetic properties of circuits with non-sinusoidal voltages. The problem gathered meaning with power electronics development. In seventies of twentieth century Shepherd and Zakikhani have presented their reactive power idea [7]. This theory is limited only to one-phase circuits. The authors have distributed current sources into two components:

$$i(t) = i_R(t) + i_r(t) \qquad (13)$$

where:

$$i_R(t) = \sqrt{2}\sum_{n=1}^{\infty}|I_n|\cos\varphi_n\cos(n\omega t + a_n) \qquad (14)$$

Resistive current component,

$$i_r(t) = \sqrt{2}\sum_{n=1}^{\infty}|I_n|\sin\varphi_n\sin(n\omega t + a_n) \qquad (15)$$

Reactive current component, where:

$a_n - \arg U_n$,

$\varphi_n$ - angle ($U_n$,$I_n$)

Whereby these currents are mutually orthogonal:

$$\int_0^T i_R(t) i_r(t) dt = 0 \tag{16}$$

The current defined by dependence (17) is called the passive reactance current and can be interpreted as current which is connected with reflexive source-receiver energy flow, and his measure is the reactive power Q.

$$i_r(t) = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} j_0 B_n U_n \exp(jn\omega t) \tag{17}$$

This current can be compensated for finite harmonics number with a reactive two-terminal network connected parallel to receiver (Figure 1) with susceptance for every examined harmonic.

This property was noticed for the first time by E. Emmanuel [3]. On the basis of Shepherd and Zakikhani's theory it was possible to determine compensatory capacitor capacitance -"optimal capacitance", for which the source coefficient is the biggest:

$$C_{opt} = \frac{\sum_{n=1}^{\infty} n|U_n||I_n|\sin\varphi_n}{\omega\sum_{n=1}^{\infty} n^2|U_n|^2} = \frac{\sum_{n=1}^{\infty} nQ_n}{\omega\sum_{n=1}^{\infty} n^2|U_n|^2} \tag{18}$$

The advantages of this method are:

- ir(t) current determination, which can be compensated for finite harmonics number using reactive two-terminal network,
- optimal capacitance value determination.

Kusters and Moore have created a theory which enables to calculate the optimal compensating capacity [6], which significantly simplifies the compensation in comparison with Shepherd and Zakikhani's theory, because of its harmonics manipulation, which requires the knowledge on phase displacement of particular harmonics.

According to this theory the current can be resolved into three orthogonal components: active current $i_a$ (identical with active current from Fryze's definition), reactive capacity current $i_{rC}$ and complementary reactive current $i_{rCs}$.

$$i = i_a + i_{rC} + i_{rCs} \tag{19}$$

Optimal compensation capacitance is then defined in the time domain as:

$$C_{opt} = -\frac{\dot{u} \circ i}{\|\dot{u}\|^2} = \frac{Q}{\left\|\frac{du}{dt}\right\|\|u\|} \tag{20}$$

Compensation made with this method is a lot easier than according to Shepherd and Zakikhani's theory. Also the capacitance values calculated according to formula (20) are identical like for dependence (18).

## IV. Reactive power compensation

In order to present the compensation effect with the help of proposed by author theories, a choke with unknown parameters was supplied with net voltage as shown on (Figure 2):

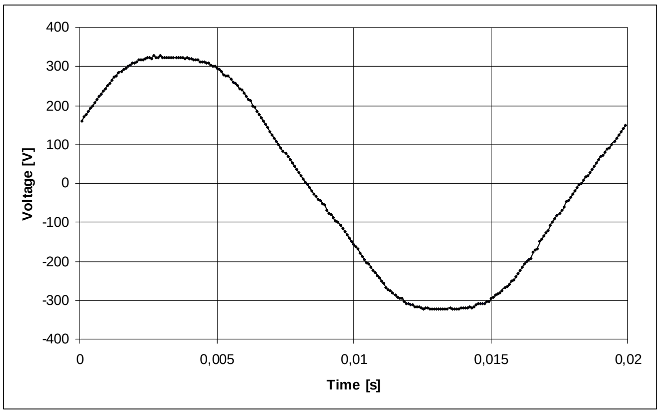


Figure 2. Voltage versus time graph

For voltage rms value $U_{rms}$=235.65 V the system response was current shown on (Figure 3) with current rms value $I_{rms}$=2.7511 A. The power factor value was PF=0.127.

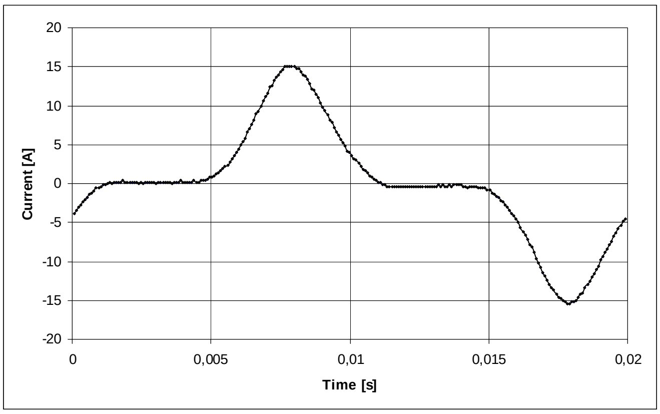


Figure 3. Current versus time graph

After making the observation of current it can be seen that the element is a strongly nonlinear object. Reactive power according to C.I. Budeanu, evaluated from (5) equation, equals Q=550.59 var. Optimal capacitance, which has to be connected to the circuit to achieve the compensation, was evaluated according to (4) equation, equals $C_{opt}$=2.40 mF. Within making the measurements this capacity was corrected depending on the value of reactive power; more and more the reactive power was closer to zero, the capacitor capacitance was determined more precisely. The current rms value was $I_{rms}$=1.3680 A and the power factor was PF=0.177.

Performing the reactive power compensation supported by the M. Iliovici [5] theory, for the same measurement conditions, the reactive power calculated from the loop area which is made by drawing the u=f(i) function (Figure 4) equals Q=546.48 var; and the optimal capacitance equals $C_{opt}$=2.34 mF. Within making the circuit compensation the capacitance value was corrected. After the circuit compensation the current rms value was $I_{rms}$=1.3601 A and the power factor was PF=0.189.

For comparison the reactive power value according to C.I. Budeanu was Q=15.422 var. This means that making the compensation according to (5) the circuit is overcompensated.

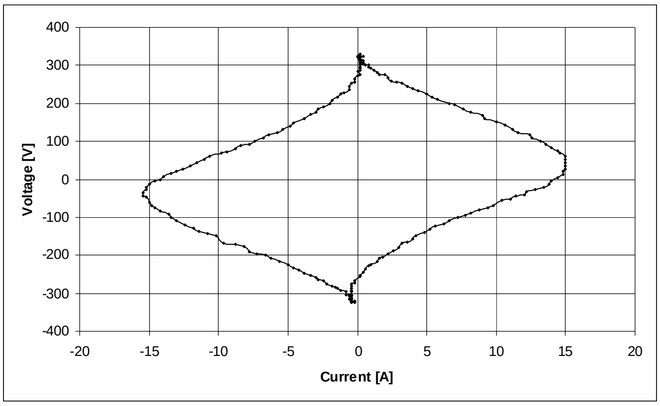


Figure 4. Reactive power loop before the compensation.

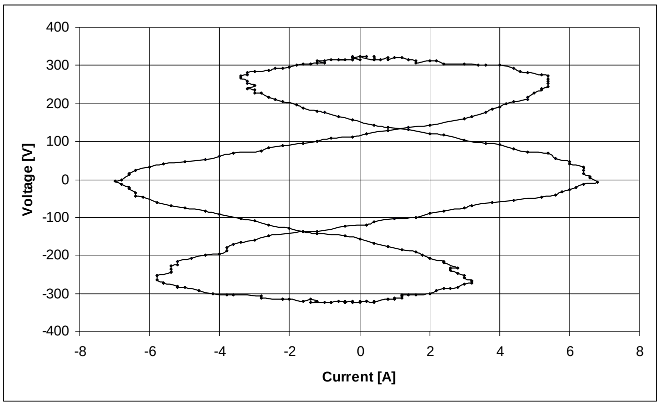


Figure 5. Reactive power loop after the compensation – loop area equals zero.

Performing the reactive power compensation supported by the Kusters and Moore theory for the same measurement conditions, the optimal capacity $C_{opt}$=36.04 uF was determined according to (20) dependence. After making the compensation the current rms value was equal $I_{rms}$=2.5925 A and the power factor was equal PF=0.128. It means that the compensation made using the dependence (20), proposed by Kusters and Moore, the circuit will be undercompensated.

## V. CONCLUSIONS

Taking advantage of mathematical dependences and modern measurement techniques we can match the compensator capacitances, without knowledge on receiver parameters. It is a very important practical feature because the compensation can be made in real-time. Thanks to this it is possible to make quick changes in capacity value and its adjustment to receiver work conditions and supplying voltage variations. The best compensation of reactive power, this means the biggest power factor is obtained taking advantage of M. Iliovici power theory. While making the compensation using the C.I. Budeanu and Kusters and Moore theory the circuit is overcompensated or undercompensated and therefore current rms value increases and the receiver operates with small power factor. Making use of power theory utilizing the loop area calculation it is possible, as shown it the article, to compensate strongly nonlinear receivers supplied with distorted signals. It is a very important practical feature.

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